Optical lattices

Hélène Perrin

Laboratoire de physique des lasers
CNRS-Université Paris 13, Sorbonne Paris Cité

Interaction of light and cold atoms
Les Houches, September 30 – October 11, 2019

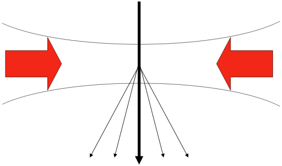
Outline of the course

- **Lecture 1: Band structure in a periodic potential**
  - Bloch theorem
  - Energy bands
  - Case of a sinusoidal potential
- **Lecture 2: Dynamics in the lattice**
  - Time of flight expansion
  - Adiabatic switching
  - Bloch oscillations
- **Lecture 3: Tight binding limit**
  - Wannier functions
  - Bose-Hubbard Hamiltonian
  - Mott transition

Standing wave along x axis plays the role of a phase mask with transmission 1.

Motion along x:

$$H = \frac{P^2}{2M} + V_0 \sin^2(kx)$$

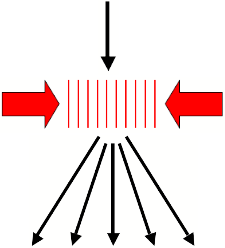


Thin grating approximation: transit time $\tau = \frac{w_0}{v}$ very short. The atoms don't move along x while crossing the light beam.

Starting from $|p_x = p_0\rangle$ and using $\sin^2 kx = [1 - \cos(2kx)]/2$:

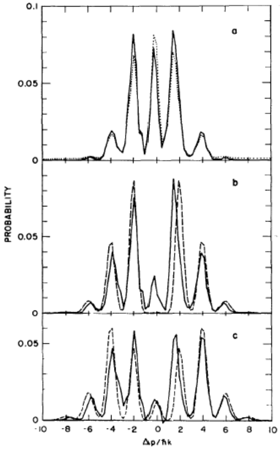
$$|\psi(\tau)\rangle = e^{iV_0\tau \sin^2(kx)/\hbar}\,|p_x = p_0\rangle = e^{i\varphi} e^{iV_0\tau\cos(2kx)/2\hbar}\,|p_0\rangle$$

$$= e^{i\varphi}\sum_n i^n J_n\left(\frac{V_0\tau}{2\hbar}\right) e^{-i2nkx}|p_0\rangle = e^{i\varphi}\sum_n i^n J_n\left(\frac{V_0\tau}{2\hbar}\right)|p_0 - 2n\hbar k\rangle$$

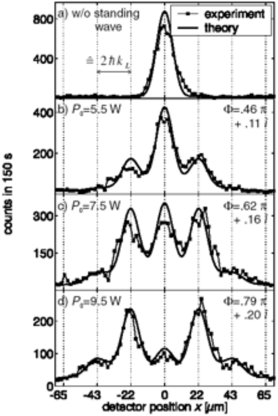


weight:

$$\left| J_n\left(\frac{V_0\tau}{2\hbar}\right)\right|^2$$



sodium atoms
Pritchard, 1985



C_{60} molecules
Zeilinger, 2001

Energy change along x (for a small angle, or $p_0 \ll \hbar k$):

$$\Delta E_x = \frac{(p_0 + 2\hbar k)^2}{2M} - \frac{p_0^2}{2M} = 2p_0 \frac{\hbar k}{M} + 4\frac{\hbar^2 k^2}{2M} \simeq 4E_{\rm rec}$$

Energy change along x (for a small angle, or $p_0 \ll \hbar k$):

$$\Delta E_x = \frac{(p_0 + 2\hbar k)^2}{2M} - \frac{p_0^2}{2M} = 2p_0\frac{\hbar k}{M} + 4\frac{\hbar^2 k^2}{2M} \simeq 4E_{\mathrm{rec}}$$

Allowed momentum change along z: $\lesssim \hbar/w_0 \ll Mv$

$\Rightarrow$ maximum energy change along z:

$$\Delta E_z \leq \frac{(Mv + \hbar/w_0)^2}{2M} - \frac{1}{2}Mv^2 \simeq \hbar v/w_0 = \frac{\hbar}{\tau}$$

(also valid in pulsed mode).

ÉCOLE DE PHYSIQUE DES HOUCHES

Diffraction by a light mask

Energy conservation

Energy change along x (for a small angle, or $p_0 \ll \hbar k$):

$$\Delta E_x = \frac{(p_0 + 2\hbar k)^2}{2M} - \frac{p_0^2}{2M} = 2p_0\frac{\hbar k}{M} + 4\frac{\hbar^2 k^2}{2M} \simeq 4E_{\rm rec}$$

Allowed momentum change along z: $\lesssim \hbar/w_0 \ll Mv$

$\Rightarrow$ maximum energy change along z:

$$\Delta E_z \leq \frac{(Mv + \hbar/w_0)^2}{2M} - \frac{1}{2}Mv^2 \simeq \hbar v/w_0 = \frac{\hbar}{\tau}$$

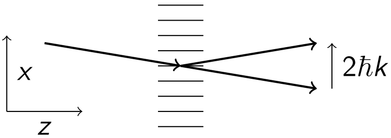
(also valid in pulsed mode).

$\Rightarrow$ possible only if $4E_{\rm rec} < \hbar/\tau$ or $\tau < \hbar/4E_{\rm rec}$

Is diffraction possible for a thick grating?

ÉCOLE DE PHYSIQUE DES HOUCHES

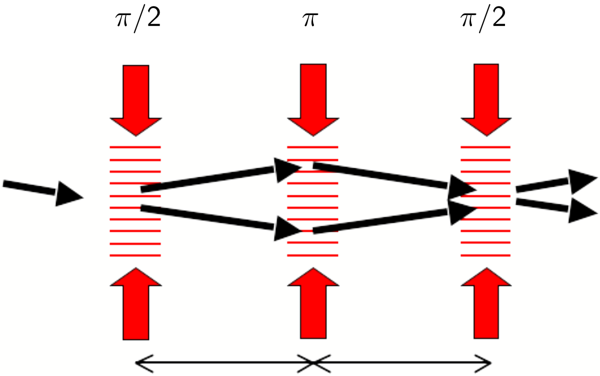
Energy and momentum are conserved for particular initial momenta satisfying $p_0 = n\hbar k$ along x: $\Rightarrow p_f = -p_i$



first order: Rabi oscillations between $|p_x = -\hbar k\rangle$ and $|p_x = +\hbar k\rangle$
$\Rightarrow$ beamsplitter with adjustable weights!

N.B. Problem on diffraction by a standing wave available upon request.

Example of use: a $\pi/2 - \pi - \pi/2$ scheme as Mach-Zehnder interferometer.



Ideal for building an inertial sensor: measurement of g or Earth's rotation.

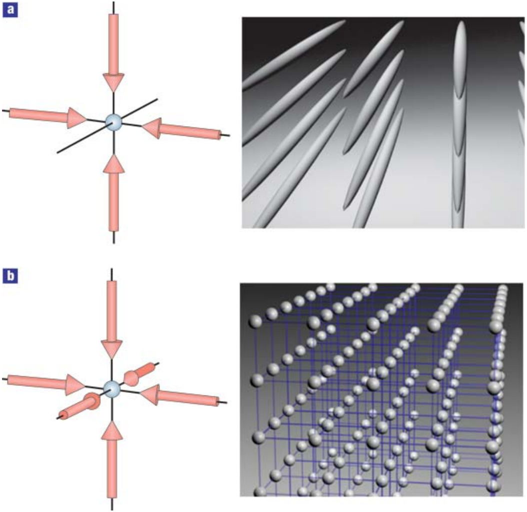
Principle of optical lattices
Square lattices

Standing waves along 1, 2 or 3 axes, with different frequencies.

2 standing waves:
2D lattice of tubes

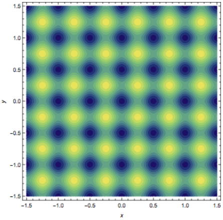
3 standing waves:
3D lattice

I. Bloch, Nat. Phys. (2005)

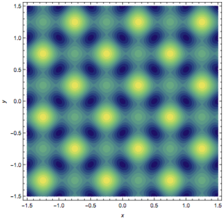


More exotic lattices with a single frequency and controlled relative phases.

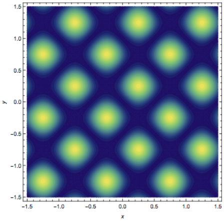
$$E(x, y) \propto \sin(kx) + e^{i\phi} \sin(ky)$$



$\phi = \pi/2$



$\phi = 2\pi/5$



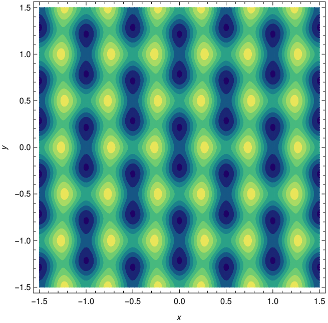
$\phi = 0$

More exotic lattices with a combination of two single frequency lattices (at ω_1 and $\omega_2 \neq \omega_1$) with controlled relative phases.

$$E_1(x,y) = \mathcal{E}_0 \cos(kx + \pi/2) \quad \text{and} \quad E_2(x,y) = \mathcal{E}_X \cos(kx) + \mathcal{E}_Y \cos(ky)$$

$$V(x,y) = -V_0 \cos^2(kx+\pi/2) - V_X \cos^2(kx) - V_Y \cos^2(ky) - 2\sqrt{V_X V_Y} \cos(kx)\cos(ky)$$

When we chose $V_0 > V_Y \gg V_X$, we get the brickwall lattice, topologically equivalent to the hexagonal lattice of graphene.



$V_0 = 2V_Y = 40V_X$

More exotic lattices with a combination of two single frequency lattices with controlled relative phases.

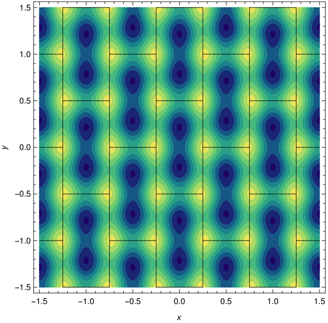
$$E_1(x,y) = \mathcal{E}_0 \cos(kx + \pi/2) \quad \text{and} \quad E_2(x,y) = \mathcal{E}_X \cos(kx) + \mathcal{E}_Y \cos(ky)$$

$$V(x,y) = -V_0 \cos^2(kx+\pi/2) - V_X \cos^2(kx) - V_Y \cos^2(ky) - 2\sqrt{V_X V_Y} \cos(kx)\cos(ky)$$

When we chose $V_0 > V_Y \gg V_X$, we get the brickwall lattice, topologically equivalent to the hexagonal lattice of graphene.

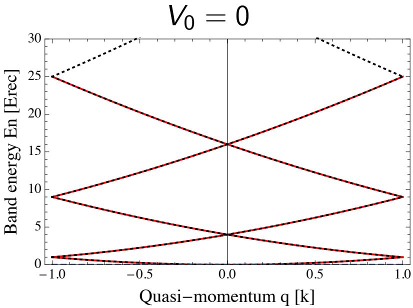
Two kinds of sites, A or B, tunnel coupled by J; almost no coupling bewteen identical sites.

Gives access to topological properties.



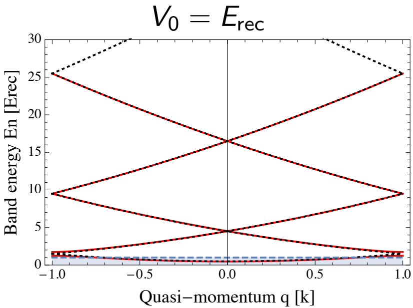
$V_0 = 2V_Y = 40V_X$

Band structure

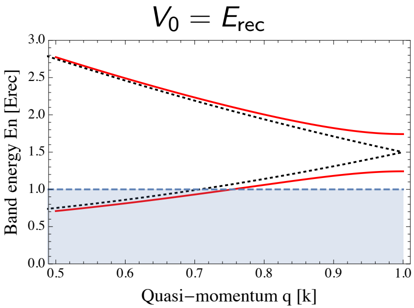


Comparison with free particle (left) or harmonic approximation (right) in dotted lines

Band structure

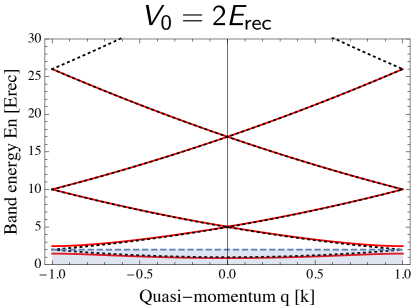


gray zone: potential depth V_0

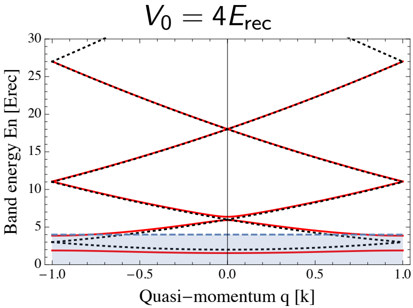


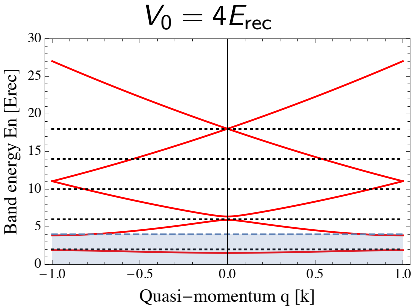
zoom around $q = 1$: gap opening

Band structure

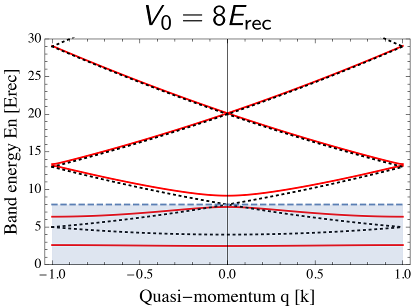


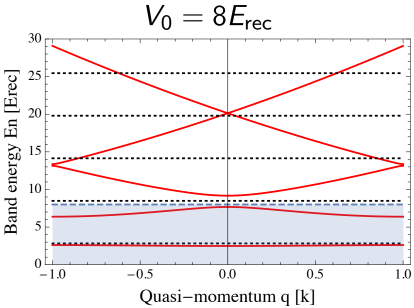
Band structure





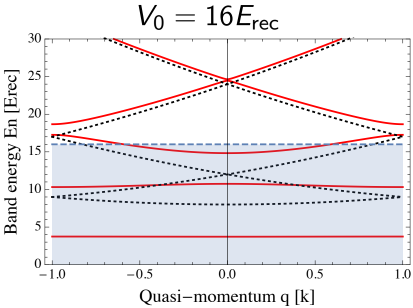
Band structure

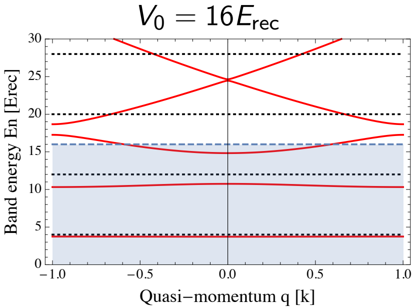




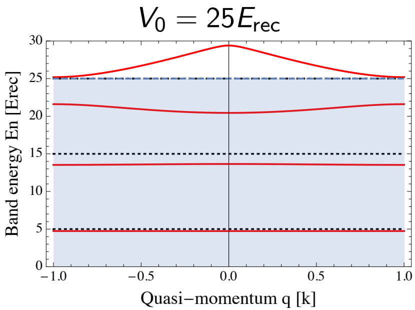
ÉCOLE DE PHYSIQUE DES HOUCHES

Band structure

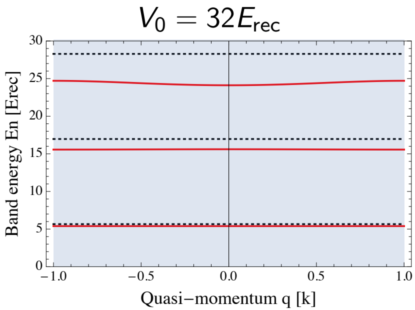




Band structure



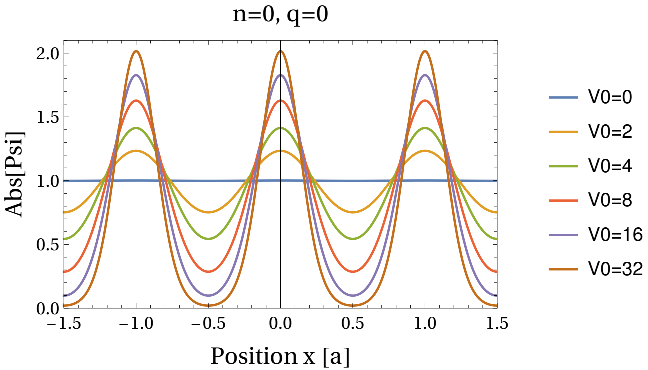
Band structure



Bloch functions

Bloch functions resemble plane waves at low V_0, and series of peaks at large V_0.

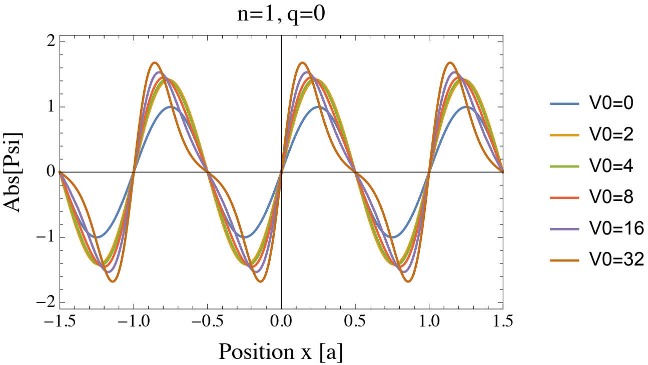
lowest band $V_0 = 0 \ldots 32 E_{\text{rec}}$



Bloch functions

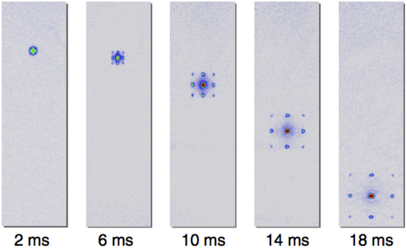
Bloch functions resemble plane waves at low V_0, and series of peaks at large V_0.

first excited band $V_0 = 0 \ldots 32 E_{\mathrm{rec}}$



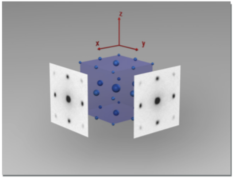
Sudden release of the optical lattice: the momentum distribution presents a periodicity $2\hbar k$.

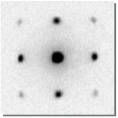
Expansion with time



Interference between the wells

bosons in a 3D lattice





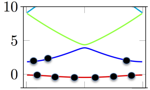
Observation along two orthogonal axes $\Rightarrow$ recover the 3D distribution

From Markus Greiner's PhD thesis.

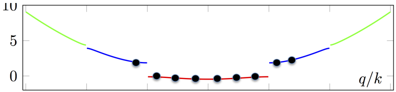
Band mapping: adiabatic release

Example: population in 2 bands

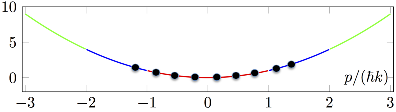
$V_0 = 4E_{\mathrm{rec}}$



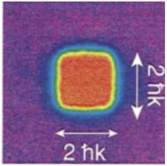
$\Leftrightarrow$



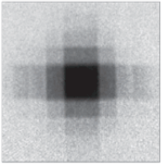
$V_0 \longrightarrow 0$



bosons in a 2D lattice
(Greiner et al. 2001)

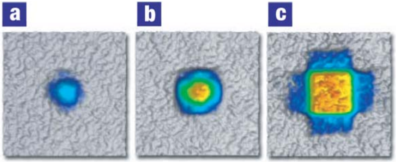


$n = 0$ only

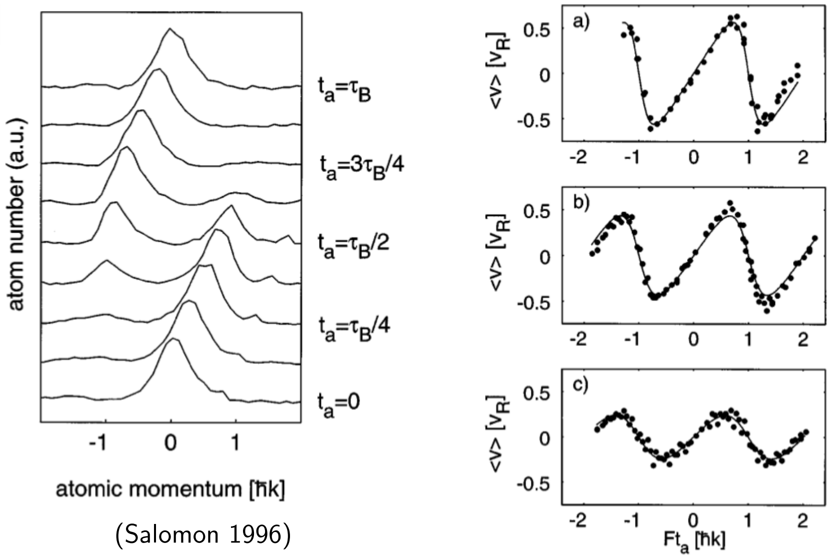


several bands

fermions in a 3D lattice
(Köhl et al. 2004)



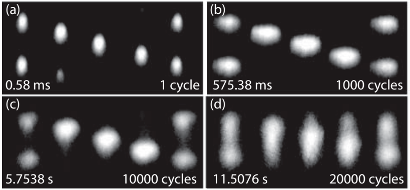
Bloch oscillations with Raman selected atoms



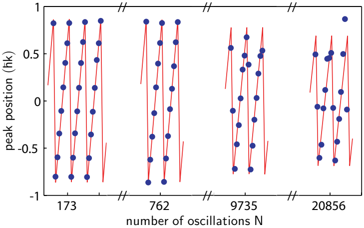
(Salomon 1996)

Bloch oscillations with a non interacting BEC

With a cesium BEC – Feshbach resonance such that $a_s = 0$.



(Nägerl 2008)

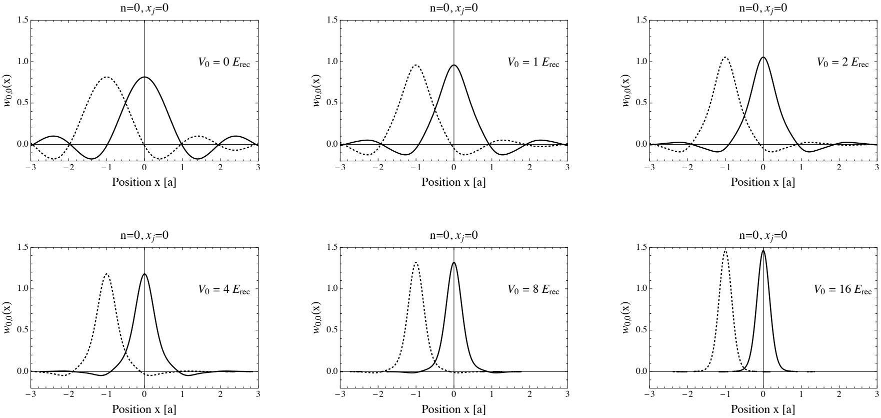


Up to 20000 oscillations!

$\Rightarrow$ use Bloch oscillations for measuring g or h/M.

Wannier functions

Wannier functions are located around a given lattice site.



ÉCOLE DE PHYSIQUE DES HOUCHES

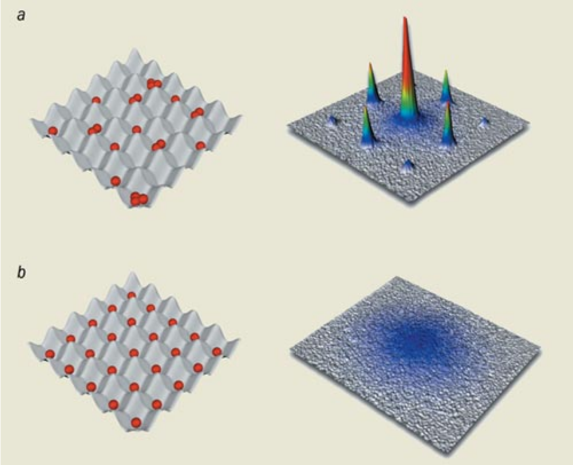
Mott transition

Observation of the Mott insulator to superfluid transition (2002):
A competition between kinetic energy and interactions

Small V_0/E_{rec}
(small U/J)

Greiner et al.,
Nature 2002

Large V_0/E_{rec}
(large U/J)



Mott shells in a lattice + harmonic trap (Greiner/Bloch 2011)