From laser cooling to Bose-Einstein condensation

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Quantum metrology and fundamental constants

Introduction
Ultra cold atoms meet quantum physics

Single particles: wave nature of matter

$$\lambda_{\mathrm{dB}} = \frac{h}{Mv}$$

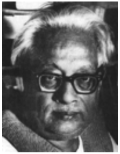
interferometry and holography, metrology, collision physics


de Broglie Schrödinger

Many particles: degenerate gases

$$n\lambda_{\mathrm{dB}}^3 > 1$$

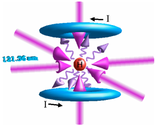
- importance of the quantum statistics (BE or FD)
- Bose-Einstein condensation, coherence, Fermi sea, superfluidity, quantum phase transitions...
- links with condensed matter physics




Introduction

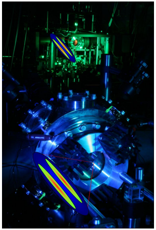
A wide range of applications:

- High precision spectroscopy (Doppler-free lines)
- Quantum information and quantum computation
- Metrology (fountains, optical clocks...)
- New insights in condensed matter physics: Bloch oscillations, superfluid-insulator transitions, Cooper pairing, search for Anderson localization... (new! observed in momentum space in Cs clouds)
- etc...

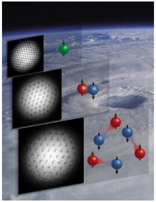


anti-hydrogen trapping (Hänsch)

micro-wave clock LNE-SYRTE Clairon / Bize / Salomon



quantum memory (Kimble)



vortex lattice (Ketterle) bosonic BEC/ fermionic BEC-BCS cross-over

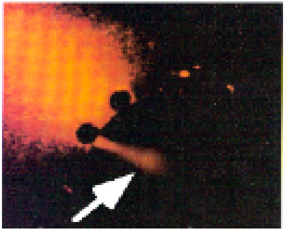
Outline

Outline of the lecture

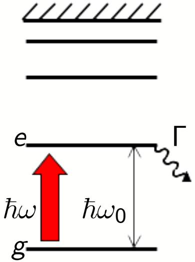
1. Light forces on atoms
2. Doppler cooling and beyond
3. Traps for neutral atoms
4. Bose-Einstein condensation

Light forces on atoms

Light forces on atoms



Atom-light interaction



Consider photons of energy $\hbar\omega$, momentum $\hbar k$, quasi-resonant with a transition between states e and g: $\hbar\omega \simeq E_e - E_g = \hbar\omega_0$.

$\Rightarrow$ two-level approximation

After each absorption/emission, the atomic velocity changes by

$$v_{\mathrm{rec}} = \frac{\hbar k}{M}$$

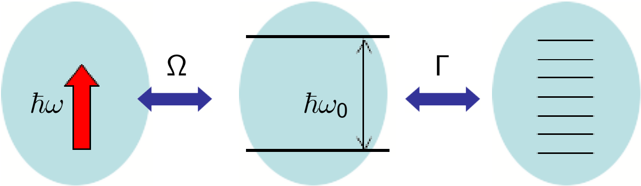
the recoil velocity, at a rate $\simeq \Gamma$.

For alkali, $3 \leq v_{\mathrm{rec}} \leq 30\,\mathrm{mm.s}^{-1}$, $\Gamma^{-1} \sim 10$ to $100\,\mathrm{ns}$.

Corresponding acceleration: $a_{\mathrm{pr}} \simeq \Gamma v_{\mathrm{rec}} \sim 10^4$ to $10^5\, g$

Atom-light interaction

3 systems in interaction : laser – atom – vacuum



The laser field $\mathbf{E}(\mathbf{r},t) = \mathcal{E}(\mathbf{r})/2 \times \left(\epsilon(\mathbf{r})e^{-i\omega t - i\phi(\mathbf{r})} + \text{c.c.}\right)$ is coupled to the atomic electric dipole moment $\mathbf{d} = \langle e|\hat{\mathbf{D}}|g\rangle$ with the Rabi frequency

$$\hbar\Omega(\mathbf{r}) = -\left(\mathbf{d}.\epsilon(\mathbf{r})\right)\mathcal{E}(\mathbf{r})$$

N.B. link with saturation intensity I_s: $\Omega^2/\Gamma^2 = I/2I_s$
Typical value of I_s: a few mW/cm^2

Light forces

The coupling term responsible for light forces is (in RWA)

$$\hat{V}_{\mathrm{laser}} = -\hat{\mathbf{D}}.\mathbf{E}(\hat{\mathbf{r}}, t) \simeq \frac{\Omega(\hat{\mathbf{r}})}{2}\left(|e\rangle\langle g|e^{-i\omega t - i\phi(\hat{\mathbf{r}})} + \mathrm{h.c.}\right)$$

The mean force acting on an atom for a given position **r** and velocity **v** is obtained in the Heisenberg picture through

$$\mathbf{F} = \langle\hat{\mathbf{F}}\rangle = \langle\frac{d\hat{\mathbf{P}}}{dt}\rangle = \frac{1}{i\hbar}\langle\left[\hat{\mathbf{P}}, \hat{H}\right]\rangle = -\langle\nabla\hat{V}_{\mathrm{laser}}\rangle$$

Two types of gradients give rise to forces: intensity gradient or phase gradient.
N.B. Fluctuation of the mean force is responsible for momentum diffusion.

Light forces

- The radiation pressure force arises from a phase gradient.
  ex: plane wave $\phi(\mathbf{r}) = -\mathbf{k}.\mathbf{r}$

$$\mathbf{F}_{\rm pr} = \hbar\mathbf{k}\frac{\Gamma}{2}\frac{s}{1+s} \qquad s = \frac{\Omega^2/2}{\delta^2+\Gamma^2/4} = \frac{I/I_s}{1+4\delta^2/\Gamma^2}$$

  s is the *saturation parameter*. $\delta = \omega - \omega_0$ is the *detuning*.

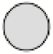
- The dipole force is due to an intensity gradient.

$$\mathbf{F}_{\rm dip} = -\frac{\hbar\delta}{2}\frac{\nabla s(\mathbf{r})}{1+s(\mathbf{r})}$$

  It derives from the dipole potential $U_{\rm dip} = \frac{\hbar\delta}{2}\ln\left(1+s(\mathbf{r})\right)$.

Radiation pressure
Physical interpretation

resonant photons



atom

$$\mathbf{F}_{\rm pr} = \hbar\mathbf{k}\frac{\Gamma}{2}\frac{s}{1+s}$$

- absorption – spontaneous emission cycles, at a rate $\gamma_{\rm fluo}$
  $$\gamma_{\rm fluo} = \frac{\Gamma}{2}\frac{s}{1+s}$$
- each changes the atomic momentum by $\hbar k = Mv_{\rm rec}$ in average

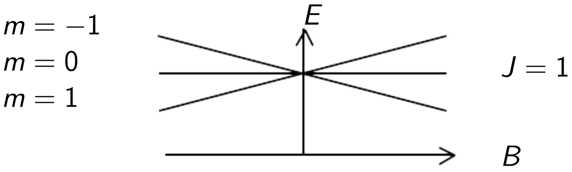
Force: $\mathbf{F}_{\rm pr} = \hbar\mathbf{k}\gamma_{\rm fluo}$ maximal value: $\mathbf{F}_{\rm pr} = \hbar\mathbf{k}\frac{\Gamma}{2}$

ex: for sodium atoms, $a \sim 10^5\, g$

$\Rightarrow$ stopping a thermal beam at $v = 100\,\mathrm{m/s}$ over 1 cm!

Zeeman effect

Reminder: Zeeman effect for a $J = 1$ spin: $\Delta E = m_J g_J \mu_B B$.

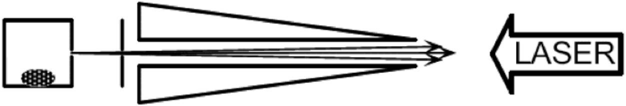


$\omega_0' = \omega_0 + \gamma B$ where $\gamma = \dfrac{g_J \mu_B}{\hbar}$

The resonance frequency is position dependent in a inhomogeneous magnetic field.

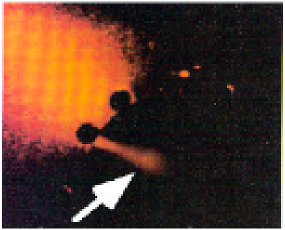
Radiation pressure
Example of application

Zeeman slower (B. Phillips et al., J. Hall et al., 1985)



oven tapered magnetic field resonant laser

such that $\omega_0(z) = \omega_0 + \gamma B(z) = \omega + kv(z)$



stopped atomic beam

Dipole force
Low saturation limit

The dipole force comes from photon redistribution inside the laser beam: stimulated emission.

- In the limit of low saturation $s \ll 1$:

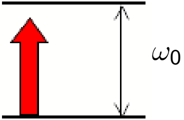
$$U_{\rm dip} = \frac{\hbar\delta}{2} \ln\left(1 + s(\mathbf{r})\right) \simeq \frac{\hbar\delta s}{2} \qquad\qquad s = \frac{\Omega^2/2}{\delta^2+\Gamma^2/4}$$

- For large detunings $|\delta| \gg \Gamma$, $s \simeq \frac{\Omega^2}{2\delta^2}$

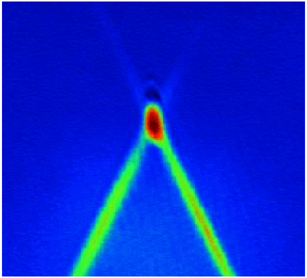
  and the dipole potential is $U_{\rm dip} \simeq \dfrac{\hbar\Omega^2}{4\delta}$
- $\delta < 0$: attractive potential $\delta > 0$: repulsive potential
- spontaneous emission rate $\gamma_{\rm fluo} \propto \Gamma\dfrac{\Omega^2}{\delta^2} \propto \dfrac{\Gamma}{\delta} U_{\rm dip}$

  $\Rightarrow$ conservative force for large detunings

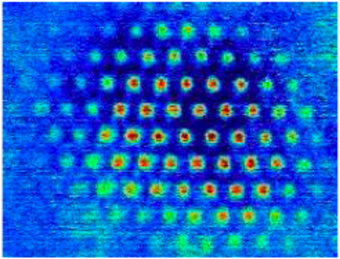
Dipole force
Negative detunings



For red detunings ($\delta < 0$), attraction to high intensity regions.



crossed dipole trap
H. Perrin, PhD thesis

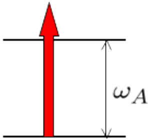


optical lattice
D. Boiron, PhD thesis

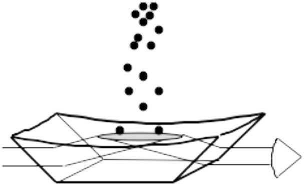
trap depth from 1 μK to several mK.

Dipole force

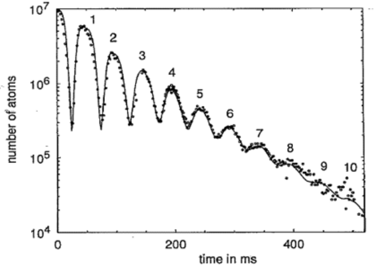
Positive detunings



For blue detunings ($\delta > 0$), repulsion from high intensity regions.



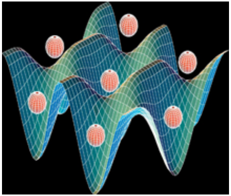
evanescent wave mirror

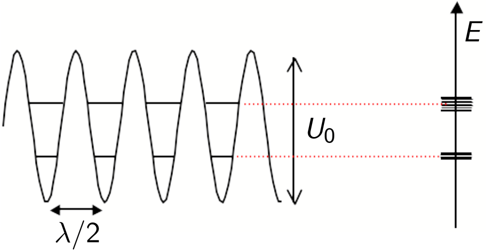


atoms bouncing off the mirror
J. Dalibard (1994)

Dipole force
Optical lattices

Optical lattices: standing waves with $\delta > 0$ or $\delta < 0$





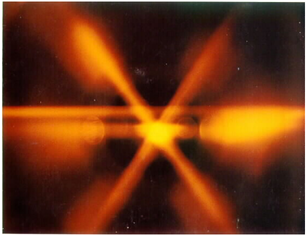
Important parameters:

- interband spacing $\hbar\omega_{\mathrm{osc}} = 2\sqrt{U_0 E_{\mathrm{rec}}}$ $\quad E_{\mathrm{rec}} = \frac{\hbar^2 k^2}{2M}$
- lowest band width / tunneling:

  $$J \propto \delta E \propto e^{-2\sqrt{U_0/E_{\mathrm{rec}}}}$$ possibly small
- effective mass in the lowest band: $M_{\mathrm{eff}} \propto 1/\delta E$ possibly large
- Lamb-Dicke regime: $\Delta x \ll \lambda \Longleftrightarrow \hbar\omega_{\mathrm{osc}} \gg E_{\mathrm{rec}}$

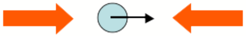
Doppler cooling and beyond

Doppler cooling and beyond

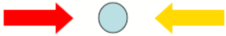


Principle of Doppler cooling

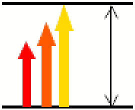
Two counter-propagating red detuned laser beams:



$+\mathbf{k}, \omega$ $\quad \mathbf{v} \quad$ $-\mathbf{k}, \omega$



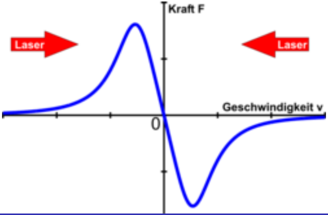
$\omega - \mathbf{k.v}$ $\qquad$ $\omega + \mathbf{k.v}$



In the low intensity limit $s \ll 1$, the radiation pressure forces add:

$$\mathbf{F} = \hbar\mathbf{k}\frac{\Gamma}{2}\left(s_+(\mathbf{v}) - s_-(\mathbf{v})\right) \qquad s_\pm(\mathbf{v}) = \frac{I/I_s}{1 + 4(\delta \mp \mathbf{k.v})^2/\Gamma^2}$$

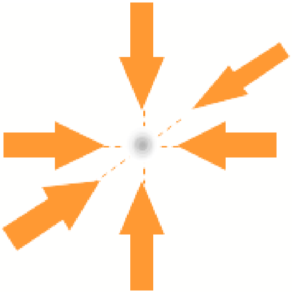
At low velocity $v \ll \Gamma/k$, one gets a friction force $\mathbf{F} = -\alpha\mathbf{v}$ with a friction coefficient $\alpha = \hbar k^2\, s_0 \frac{-2\delta\Gamma}{\delta^2+\Gamma^2/4}$

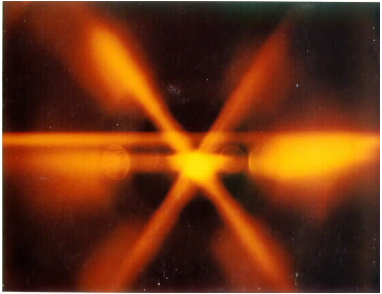


$\alpha > 0$ for $\delta < 0$
$\alpha_{\max} = 2\hbar k^2\, s_0$ for $\delta = -\Gamma/2$
damping time $\sim \hbar/E_{\mathrm{rec}}$:
a few 100 μs for Rb @ $s = 0.1$

3D molasses

Generalization to 3D:





first Na molasses at NIST

Limit temperature in Doppler cooling

Fluctuation of the cooling force due to random direction of spontaneous emission

$\Rightarrow$ Competition cooling vs random walk in momentum space

diffusion coefficient $D_p = \hbar^2 k^2 \Gamma s_0$

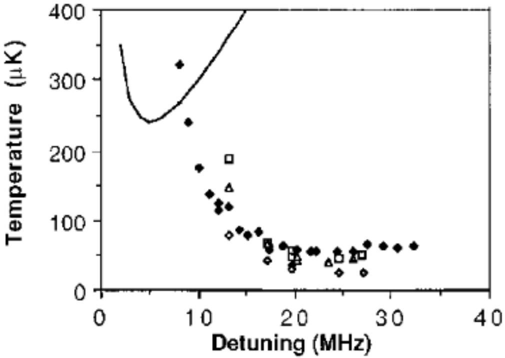
limit temperature : $k_B T_D = \dfrac{D_p}{\alpha} > \dfrac{D_p}{\alpha_{\max}} = \dfrac{\hbar\Gamma}{2}$ for $\delta = -\Gamma/2$

T_D is the Doppler temperature

ex: $T_D = 240\,\mu$K for sodium, $T_D = 125\,\mu$K for cesium

What about experiments?

First temperature measurements

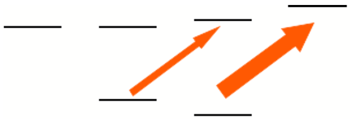


Bill Phillips, 1988

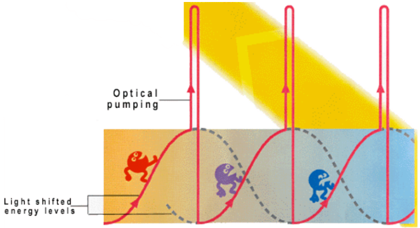
The measured temperature is lower than expected!
The scaling with laser detuning is also different.

Sisyphus cooling

Missing ingredients: polarization gradients and internal atomic structure.



Optical pumping correlated with potential depth.



Limit temperature $k_B T \propto I/\delta$.
Fundamental limit for Sisyphus cooling: the recoil temperature.
Typical value: $T_{\mathrm{rec}} = 2E_{\mathrm{rec}}/k_B = 2.4\,\mu\mathrm{K}$ for Na, $200\,\mathrm{nK}$ for Cs.

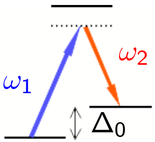
Subrecoil cooling

Is it possible to beat the recoil limit? YES!

Recipe for subrecoil laser cooling:

1. Protect atoms with very low velocity from laser light: create a dark state
2. Excite other atoms: random walk in momentum space to accumulate atoms into the dark state

Example: Raman cooling: counter-propagating ω_1 and ω_2



Narrow 2-photon velocity selective transition:

$\omega_1 - \omega_2 - \Delta_0 = 2kv + 4\frac{E_{rec}}{\hbar}$

$\Rightarrow$ excite only rapid atoms...

Laser cooling of ions

Ions also may be cooled to very low temperature, through sideband cooling.

- Ions may be strongly confined thanks to their charge, in both states g and e, in the same harmonic trap, with $\omega_{\rm osc}$ in the MHz range.
- One reaches the Lamb-Dicke limit $\Delta x \ll \lambda$, or $\hbar\omega_{\rm osc} \gg E_{\rm rec}$
- In this regime, spontaneous emission preserves the external harmonic level n: $|e, n\rangle \rightarrow |g, n\rangle$
- Pumping on the red sideband $|g, n\rangle \rightarrow |e, n-1\rangle$

Laser cooling of ions

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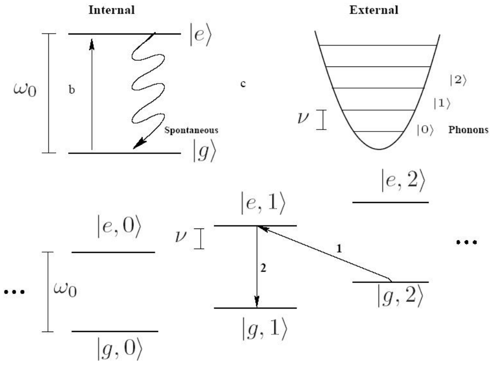
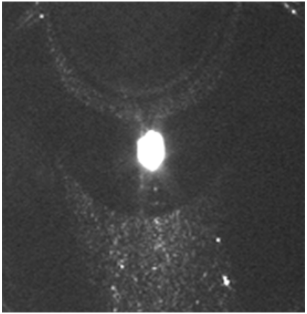


$\Rightarrow$ cooling to harmonic ground state

Traps for neutral atoms

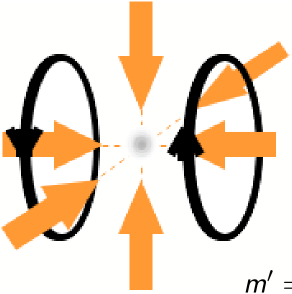
Traps for neutral atoms

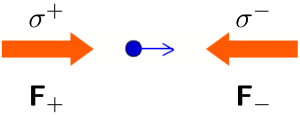


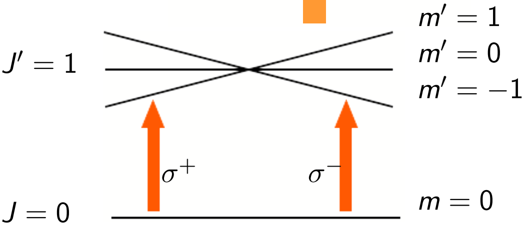
a rubidium MOT at LPL

The magneto-optical trap

Adding a magnetic gradient b' to the molasses configuration results in atom trapping.
The beam polarizations are of opposite circulation and create a radiation pressure imbalance.







$\Rightarrow$ position dependent detuning $\delta \pm \mu b' x$

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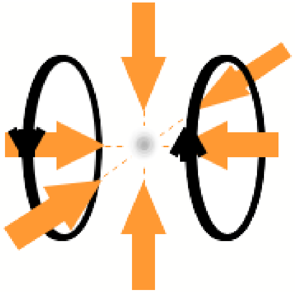


Resulting force at low saturation: $\mathbf{F} = -\alpha \mathbf{v} - \kappa \mathbf{r}$

$$\kappa = \mu b' s_0 \frac{-2\delta\Gamma}{\delta^2 + \Gamma^2/4}$$ expression similar to α

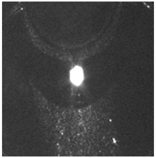
A MOT can be loaded from a Zeeman slower or directly from a vapour.

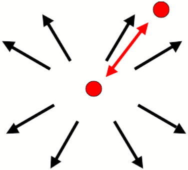
The magneto-optical trap

Typical figures:

- easy to implement and widely used for alkali – less easy for alkaline earth, etc
- temperature $T \sim 100\,T_{\rm rec}$ limited by the magnetic gradients
- density $n \sim 10^{10}\,\mathrm{cm}^{-3}$ typically, limited by photon reabsorption (multiple photon scattering)
- loading time between 0.1 and 10 s, depending on the background pressure

⇒ an ideal starting point for many experiments!





$1/r$ repulsive force

Conservative traps

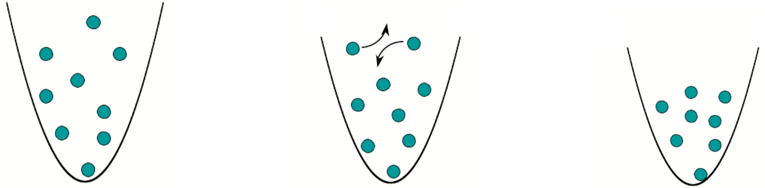
The MOT is a dissipative trap. The phase space density in a MOT is limited to 10^{-7}–10^{-6} typically.
Random photon scattering limits its performances in this regards.
To cool further: evaporative cooling in a conservative trap is generally used. Two kinds of conservative traps for neutral atoms:

- *optical dipole traps* are conservative for a large enough detuning
- *magnetic traps* are most commonly used for reaching quantum degeneracy of atoms polarized in their upper spin state

$$V(\mathbf{r}) = \mu|B(\mathbf{r})| \text{ where } |B| \text{ has a local non zero minimum}$$

Evaporative cooling
Principle

Evaporative cooling occurs through rethermalization after filtering the "hottest" particles.



equilibrium
at T_1

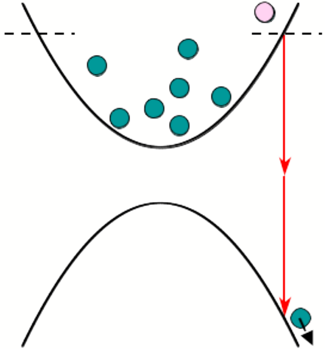
decrease trap depth
+ elastic collisions

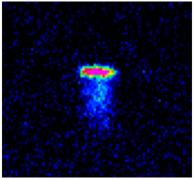
new equilibrium
at $T_2 < T_1$

Evaporative cooling

Implementation in a magnetic trap

Radio-frequency evaporative cooling: the RF field resonantly out-couples atoms from the trap at a given location.



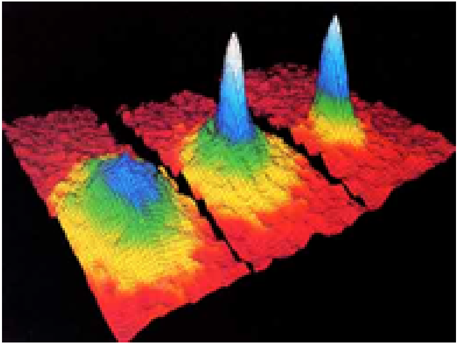


atoms evaporated from the magnetic trap, falling due to gravity (LPL)

N.B. Evaporative cooling may also be implemented in an optical trap by lowering the laser intensity.

Bose-Einstein condensation

Bose-Einstein condensation



first BEC, Cornell/Wieman, JILA (1995)

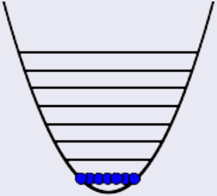
Bosons and fermions

Reminder: bosons and fermions at low temperature.

bosons

Bose-Einstein statistics

$$f(E) : \frac{1}{\frac{1}{z}e^{E/k_B T} - 1}$$

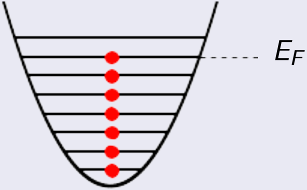


$$f(E \neq 0) \xrightarrow[T \to 0]{} 0$$

fermions

Fermi-Dirac statistics

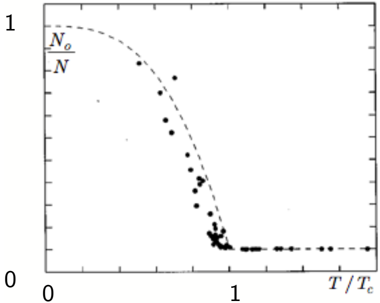
$$f(E) : \frac{1}{\frac{1}{z}e^{E/k_B T} + 1}$$



$0 \leq f \leq 1$ at most 1

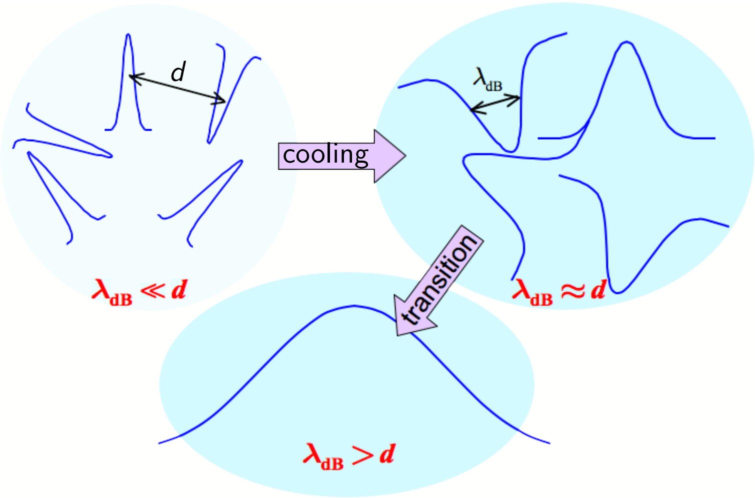
Critical temperature

For a given atom number N, the number of atoms in the excited states $N_{\rm ex}$ is limited and below a critical temperature T_c, the occupancy of the ground state becomes macroscopic ($\sim N$).



Bose-Einstein condensation is a quantum phase transition directly linked to bosonic statistics.
T_c is determined by $n\lambda_{\rm dB}^3 \sim 1$ where $\lambda_{\rm dB} = h/\sqrt{2\pi M k_B T}$ is the thermal de Broglie wavelength.

Critical temperature

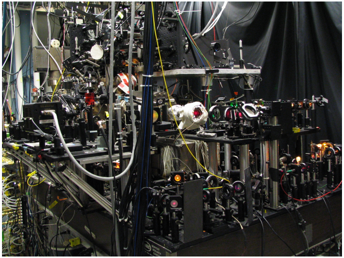


at $n\lambda_{\rm dB}^3 \sim 1$: a single wave function for all the atoms!

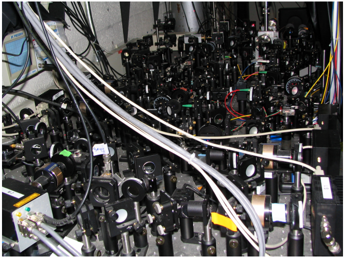
BEC production

- Typical transition temperature: a few 100 nK for 10^6 atoms in a magnetic trap.
- Combine laser cooling and evaporative cooling
- Ultra high vacuum required (10^{-12} mbar)

as a result: quite complex experiments...



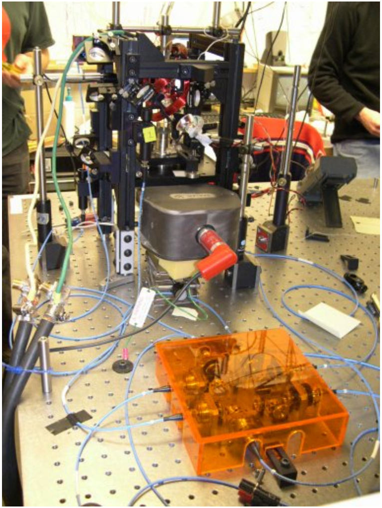
Wolfgang Ketterle
(Li experiment, vacuum)



Wolfgang Ketterle
(Li experiment, lasers)

BEC production

…which may be compacted if needed…

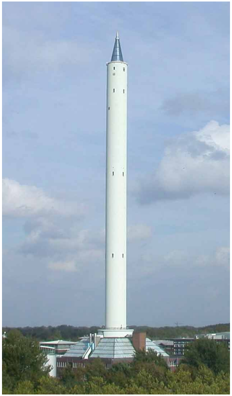


Dana Anderson's setup

BEC production

...for example for low gravity or space experiments!

an atom chip (Paris)

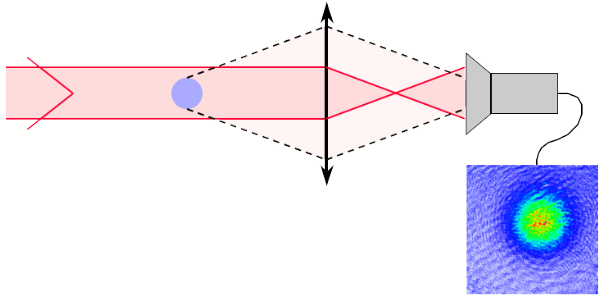


Bremen drop tower (german collaboration)

BEC detection

Onset of BEC: strong population increase of a low momentum state.

Detection by absorption imaging in time of flight experiments.



in situ images: position distribution in the trap

TOF images: velocity distribution

BEC detection

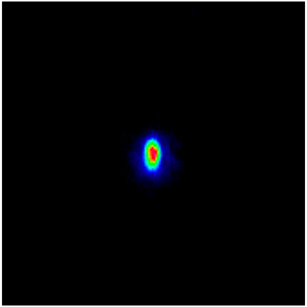
Onset of BEC: strong population increase of a low momentum state.
Detection by absorption imaging in time of flight experiments.
Anisotropy in the ballistic expansion from a cigar shaped trap:

thermal gas:

$$\Delta p_x = \Delta p_y = \sqrt{M k_B T}$$

BEC:

$$\Delta p_x = \frac{\hbar}{2\Delta x} \neq \Delta p_y$$

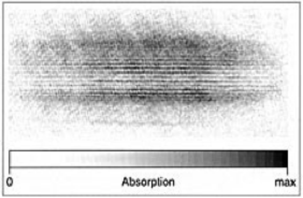


BEC @ LPL

BEC coherence

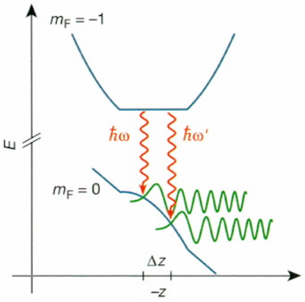
Atoms in a BEC occupy the same state ⇒ phase coherence over the whole cloud.

First evidence: interference between 2 BECs (Ketterle, MIT)

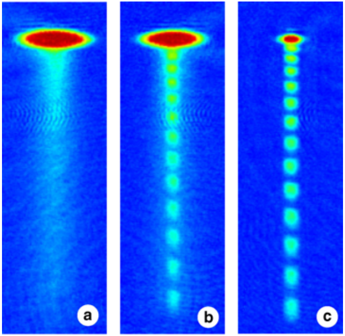


BEC coherence

Atoms in a BEC occupy the same state $\Rightarrow$ phase coherence over the whole cloud.
Further demonstration: beat note between two atom lasers (Esslinger/Bloch, Munich) $\Rightarrow$ coherence length = BEC size



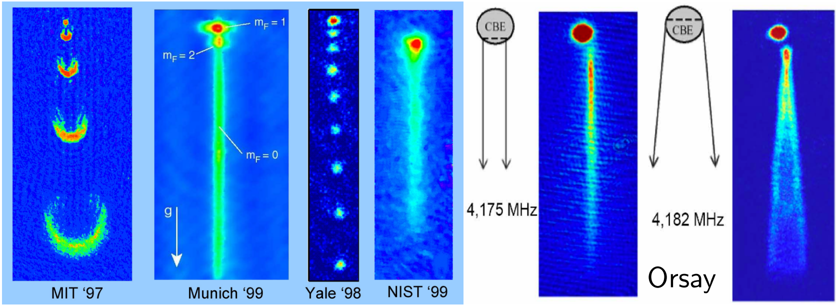
RF outcoupling



$T > T_c$ $T < T_c$ $T \ll T_c$

Atom laser

Bose-Einstein condensates may be considered as an atom laser.
Outcoupling may be pulsed or continuous:



MIT '97 Munich '99 Yale '98 NIST '99 Orsay

Ideal for atom interferometry... once interaction issues have been addressed!
N.B. Low dimensionality affects the coherence properties.

Interactions in a BEC

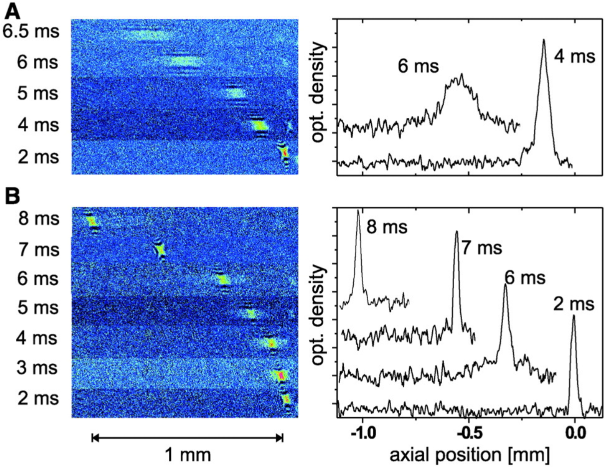
Interactions play an important role in the physics of BEC and degenerate Fermi gases. A BEC is described by the Gross-Pitaevskii equation:

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2M}\triangle\psi + V(\mathbf{r})\psi + \frac{4\pi\hbar^2 a}{M}|\psi|^2\psi$$

The scattering length a describes the interaction strength:
$a > 0$: repulsive interactions, stable trapped BEC
$a < 0$: attractive interaction, collapse.
This non linear Schrödinger equation was introduced for describing superfluid helium. A Bose-Einstein condensed dilute gas is a superfluid.
$\Rightarrow$ critical velocity, solitons, vortices, persistent current...

BEC as a superfluid

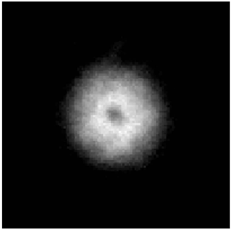
A bright soliton: dispersion balanced by attractive interactions



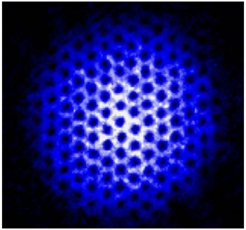
Christophe Salomon, ENS Paris

BEC as a superfluid

Abrikosov vortex lattices:



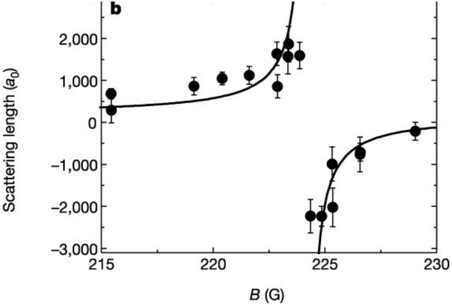
single vortex:
angular momentum $= \hbar$
(Dalibard)



vortex lattice
(Cornell)

Interactions in a BEC

Interactions may be controlled at will for most species, in sign and amplitude, by tuning the magnetic field through Feshbach resonances:

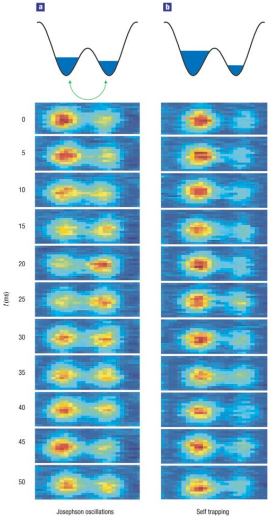


Feshbach resonance in fermionic ^{40}K (D. Jin)

Controlling the interactions allows the creation of molecular BECs and the Cooper pairing of fermions to obtain a fermionic superfluid.

BEC in a double well

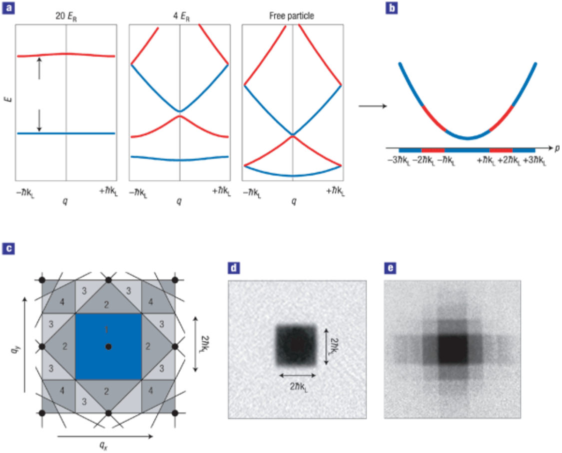
When interactions compete with tunneling between two wells, one observes either Josephson oscillations or self-trapping:



Oberthaler (2005)

BEC in optical lattices

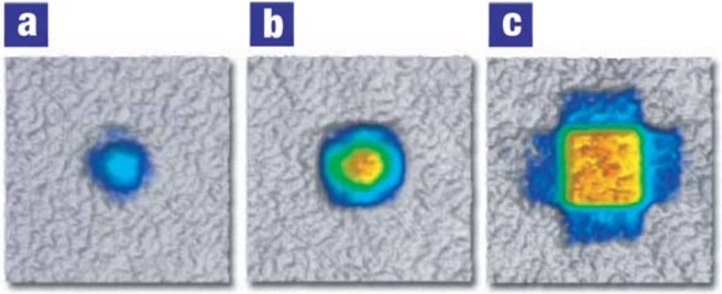
BEC in optical lattices: Bloch band mapping



(Immanuel Bloch)

BEC in optical lattices

Fermions: imaging the Fermi sea:



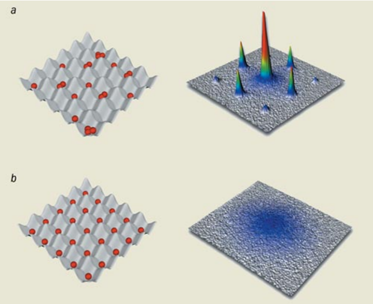
(Tilman Esslinger)

BEC in optical lattices

A quantum phase transition: Mott insulator to superfluid transition.
⇒ From a number state to a phase state.

superfluid
(phase state)

Mott insulator
(number phase)



(Bloch 2002)

The Mott insulator state is a highly correlated state, useful for molecular production or quantum computing...

BEC: latest news

Since the first observation of BEC in dilute gases in 1995, the field of ultracold atoms – bosons or fermions – gives rise to a number of major achievements, and is still growing.
Latest news from BEC 2007 conference:

- observation of persistent current
- Kosterlitz-Thouless transition in two different systems
- BEC in high finesse cavities
- spin textures in BECs
- number squeezing in a double well system
- double particle tunneling in optical lattices
- fermionic pairing for unequal spin populations...

Further reading

- On laser cooling and trapping:
  – C. Cohen-Tannoudji: *Atomic motion in laser light*, in *Fundamental systems in Quantum optics*, Les Houches, Session LIII (Elsevier, 1992).
  – H. J. Metcalf and P. van der Straten, *Laser Cooling and Trapping*, (Springer, New York, 1999)
- On BEC:
  – *Coherent Matter Waves*, Les Houches, Session LXXII, R. Kaiser, C. Westbrook and F. David Eds., (EDP Sciences; Springer, Berlin, 2001)
  – W. Ketterle, D.S. Durfee, and D.M. Stamper-Kurn: *Making, probing and understanding Bose-Einstein condensates*, In Proceedings Enrico Fermi School, Course CXL (IOS Press, Amsterdam, 1999)
  – F. Dalfovo, S. Giorgini, L. Pitaevskii and S. Stringari, Rev. Mod. Phys. **71**, 463 (1999)
  – C. J. Pethick and H. Smith, *Bose-Einstein Condensation in Dilute Gases* (Cambridge University Press, Cambridge 2001).
  – Lev P. Pitaevskii and S. Stringari, *Bose-Einstein Condensation* (Clarendon Press, Oxford, 2003).
- On BEC in optical lattices:
  – Immanuel Bloch, *Ultracold quantum gases in optical lattices*, Nature Physics **1**, 23 - 30 (2005)