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Hélène Perrin and Pierre Desbiolles

Problem: Sisyphus effect at low saturation on a $\mathbf{J_g = 1/2 \leftrightarrow J_e = 3/2}$ transition

1 Model system

In this problem, we study the Sisyphus effect for an atom with two Zeeman levels in the ground state ($J_g = 1/2$), in the presence of a strong polarisation gradient of light. We consider a transition $J_g = 1/2 \leftrightarrow J_e = J_g + 1 = 3/2$, the Clebsch-Gordan coefficient being given below. The light intensity is low, so that the saturation parameter is small ($s \ll 1$).

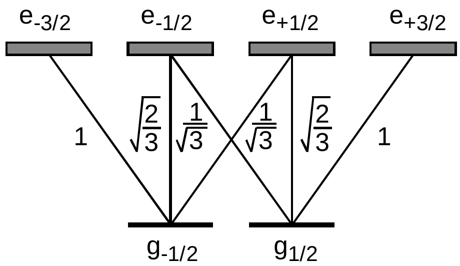

Figure 1: *1/2 ↔ 3/2 transition.*

1.1 Laser field configuration

The laser field consists of two plane waves propagating in opposite directions along the z axis. Their polarisation are orthogonal, along $\mathbf{e}_x$ and $\mathbf{e}_y$), and their amplitude are equal. More specifically, we write the two fields as:

$$\mathbf{E}_+ = \frac{\mathcal{E}_0}{2}\,\mathbf{e}_x \left(e^{-i\omega t + ik_L z} + c.c.\right) \quad \text{and} \quad \mathbf{E}_- = -\frac{\mathcal{E}_0}{2}\,\mathbf{e}_y \left(ie^{-i\omega t - ik_L z} + c.c.\right) \tag{1}$$

1. Write the total field $\mathbf{E}_L(z,t)$ as:

$$\mathbf{E}_L(z,t) = \mathbf{E}_L^+(z)\mathrm{e}^{-i\omega t} + \mathrm{c.c.} \tag{2}$$

where $\mathbf{E}_L^+(z) = \mathcal{E}_L \boldsymbol{\epsilon}(z)/2$, with $\mathcal{E}_L = \sqrt{2}\mathcal{E}_0$, and the polarisation $\boldsymbol{\epsilon}$ is given by $\boldsymbol{\epsilon}(z) = \cos kz\ \boldsymbol{\epsilon}_- - i \sin kz\ \boldsymbol{\epsilon}_+$. The polarisations $\boldsymbol{\epsilon}_+$ and $\boldsymbol{\epsilon}_-$ are the standard basis vectors corresponding to σ^+ and σ^-:

$$\boldsymbol{\epsilon}_\pm = \mp\frac{1}{\sqrt{2}}(\mathbf{e}_x \pm i\mathbf{e}_y) \tag{3}$$

2. Make a diagram illustrating the variation of the polarisation as a function of z.

1.2 Dipole force

The dipole force $\mathcal{F}_{\text{dip}}$ is given by:

$$\mathcal{F}_{\text{dip}} = -\sum_{\alpha} (\nabla E_\alpha) \Pi_\alpha \tag{4}$$

where Π_α is the population in the eigenstate $|g_\alpha\rangle$ of energy E_α of the effective hamiltonian:

$$H_{\text{eff}} = \frac{\hbar \Omega_1^2/4}{\delta^2 + (\Gamma^2/4)} \delta \Lambda(\mathbf{r}) = \hbar \delta' \Lambda(\mathbf{r}). \tag{5}$$

The total Rabi frequency (for the two beams) is defined as $\hbar\Omega_1 = -\mathcal{D}\mathcal{E}_L$ and the operator $\Lambda(\mathbf{r})$ is:

$$\Lambda(\mathbf{r}) = \left(\boldsymbol{\epsilon}^\star(\mathbf{r}) \cdot \hat{\mathbf{d}}^-\right)\left(\boldsymbol{\epsilon}(\mathbf{r}) \cdot \hat{\mathbf{d}}^+\right) \tag{6}$$

where $\hat{\mathbf{d}}$ is the reduced dipole operator (see the additional information file).

1. What is the effect of the operator $\left(\boldsymbol{\epsilon}(\mathbf{r}) \cdot \hat{\mathbf{d}}^+\right)$ on the states $|g, +1/2\rangle$ and $|g, -1/2\rangle$?
2. Show that in the case $J_g = 1/2$ the states $|g, \pm 1/2\rangle$ are the eigenstates of $\Lambda(\mathbf{r})$, and give the corresponding eigenvalues.
3. Give the expression of $\mathcal{F}_{\text{dip}}$ as a function of $\Pi_{\pm 1/2}$ and $E_{\pm 1/2}$, populations and energies of the states $|g, \pm 1/2\rangle$.

1.3 Dissipative force

Explain why the dissipative force is zero for an atom with zero velocity, for any internal substate of the groundstate.

2 Dynamics of the internal degrees of freedom

2.1 Light shifts in the ground state

1. Show that the light shifts of the groundstates $|g, \pm 1/2\rangle$ are given by:

   $$E_{+1/2}(z) = U_0 \left(-\frac{3}{2} + \cos^2 kz\right) \quad \text{and} \quad E_{-1/2}(z) = U_0 \left(-\frac{3}{2} + \sin^2 kz\right) \tag{7}$$

   where $U_0 = -2\hbar\delta s_0/3$ and s_0 is the saturation parameter for one beam. Give a graph of $E_{\pm 1/2}$ as a function of z.
2. Deduce the mean force $\mathcal{F}$ as a fonction of the population difference $\mathcal{M}(z) = \Pi_{+1/2}(z) - \Pi_{-1/2}(z)$.

2.2 Optical pumping rate

We now need to calculate $\mathcal{M}(z)$. The population in the two groundstates $|g,+1/2\rangle$ and $|g,-1/2\rangle$ evolve due to optical pumping via the excited states.

1. The departure rates from the states $|g,\pm1/2\rangle$ are given by:

$$\Gamma'_{\pm1/2}(z) = \Gamma' \langle g,\pm1/2|\Lambda|g,\pm1/2\rangle \tag{8}$$

   where $\Gamma' = \Gamma s_0$.

   On the other hand, the arrival rates to $|g,\pm1/2\rangle$ are:

$$\Gamma''_{\pm1/2}(z) = \Gamma' \langle g,\pm1/2| \sum_{q=-1,0,+1} (\boldsymbol{\epsilon}_q^\star \cdot \hat{\mathrm{d}}^-) \left(\boldsymbol{\epsilon}(\mathbf{r}) \cdot \hat{\mathrm{d}}^+\right) \sigma_{gg} \left(\boldsymbol{\epsilon}^\star(\mathbf{r}) \cdot \hat{\mathrm{d}}^-\right) (\boldsymbol{\epsilon}_q \cdot \hat{\mathrm{d}}^+)|g,\pm1/2\rangle \tag{9}$$

   in the standard basis where $\boldsymbol{\epsilon}_0 = \boldsymbol{\epsilon}_z$ (π polarisation) and $\boldsymbol{\epsilon}_{\pm1} = \boldsymbol{\epsilon}_\pm$.

   Calculate the departure and arrival rates for the two groundstates.

2. Deduce that $\mathcal{M}(z)$ obeys the following differential equation:

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{M}(z) = -\frac{1}{\tau_P}[\mathcal{M}(z) + \cos 2kz] \tag{10}$$

   where $\tau_P = 9/(2\Gamma s_0)$. What is the physical meaning of this time?

3. Determine the populations $\Pi_{\pm1/2}(z)$ in the steady state. Indicate the result on the graph of the light shifts as a function of z.

4. Give a qualitative explanation of Sisyphus cooling in the limit where the kinetic energy is much larger than the potential depth ($Mv^2/2 \gg U_0$), and the atomic velocity is such that $\Gamma \gg kv \gg \Gamma'$. Give an order of magnitude of the limit temperature T that can be obtained in this large detuning limit. How does T depend on the laser intensity and detuning?

3 Cooling mechanism for a moving atom

3.1 Characteristic times

1. What is the characteristic time for the evolution of the internal variables of the atoms t_{int}.

2. To evaluate the characteristic time for the external degrees of freedom, we assume that the atomic energy is low enough for the atom to be trapped in a well. What is then the time t_{ext} associated to the oscillations in the well?

3. We assume by now that $t_{\mathrm{int}} \ll t_{\mathrm{ext}}$ (hoping regime). Explain why the dynamics can be explained in a simplified way.

3.2 The hoping regime

1. We assume now that the atomic position is linked to time by $z = vt$ on the timescale of optical pumping. Why is this assumption justified?

2. Give the differential equation for $\mathcal{M}(t)$. Find the forced oscillation solution, using the critical velocity $v_c = 1/(2k\tau_P)$.

3. Determine the mean force $\overline{\boldsymbol{\mathcal{F}}_z(v)}$ averaged over a spatial period $\lambda/2$. Show that it is a friction force at low velocities (low with respect to what?).

4. Compare the friction coefficient α_S with the one which appears in Doppler cooling α_D.

3.3 Equilibrium temperature

1. Recall the diffusion coefficient in momentum space associated to spontaneous emission D_R at low saturation ($s_0 \ll 1$).

2. The diffusion coefficient in momentum space associated to the dipole force is:

$$D_{\text{dip}} = 2\hbar^2 k^2 \frac{\delta^2}{\Gamma} s_0 \sin^4(2kz) \tag{11}$$

   Give its spatial average.

3. Compare the two D coefficients at large detuning $|\delta| \gg \Gamma$. Deduce the limit temperature for Sisyphus cooling.

4. Check that the condition on the velocity for considering the hoping regime is compatible with the low velocity limit to get a linear friction force $\overline{\boldsymbol{\mathcal{F}}_z(v)}$.