

# Lecture 3: Superfluid dynamics at the bottom of a bubble trap

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Vortex Dynamics, Turbulence and Related Phenomena  
in Quantum Fluids — Natal, June 24-28, 2019



# Outline of the course

- Lecture 1: Bose-Einstein condensation, superfluid hydrodynamics and collective modes
- Lecture 2: Adiabatic potentials for confining quantum gases
- **Lecture 3: Superfluid dynamics at the bottom of a bubble trap**

# Tuning quantum gases

Quantum gases benefit from a wide range of tunable parameters:

- temperature in the range 10 nK – 1  $\mu$ K
- interaction strength: scattering length  $a$
- dynamical control of the confinement geometry
- periodic potentials (optical lattices)
- low dimensional systems accessible (1D, 2D)
- several internal states or species available
- easy optical detection

# Collective modes as a probe of the system

**This lecture:** exploring the collective modes at the bottom of the bubble trap:

- support **temperature** in the range  $10 \text{ nK} - 1 \mu\text{K}$
- **interaction strength** controlled by confinement
- **dynamical control of the confinement geometry**
- periodic potentials (optical lattices)
- **low dimensional systems** accessible (1D, 2D)
- several internal states or species available
- **easy optical detection**

# Outline of the course

- 1 The two-dimensional Bose gas
- 2 Overview of the collective modes
- 3 The monopole mode as a probe of the Equation Of State
- 4 The scissors mode as a probe of superfluidity
- 5 Summary & prospects

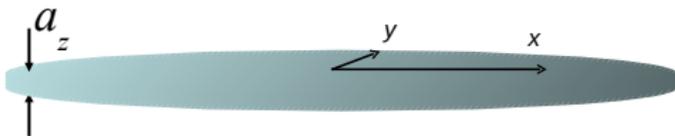
# References for the lecture

- 1 F. Dalfovo, S. Giogini, L. Pitaevskii and S. Stringari, Rev. Mod. Phys. **71**, 463 (1999).
- 2 I. Bloch, J. Dalibard, W. Zwerger, Rev. Mod. Phys. **80**, 885 (2008)
- 3 R. Dubessy et al., New J. Phys. **16**, 122001 (2014)
- 4 M. Olshanii, H. Perrin, and V. Lorent, Phys. Rev. Lett. **105**, 095302 (2010)
- 5 D. Guéry-Odelin and S. Stringari, Phys. Rev. Lett. **83**, 4452 (1999)
- 6 K. Merloti et al., Phys. Rev. A **88**, 061603(R) (2013)
- 7 M. Holten et al., Phys. Rev. Lett. **121**, 120401 (2018)
- 8 C. De Rossi et al., New J. Phys. **18**, 062001 (2016)

# Confining to two dimensions

Prepare a 2D Bose gas

- Harmonic trap  $V_{\text{ext}}(x, y, z) = \frac{1}{2}M\omega_x^2x^2 + \frac{1}{2}M\omega_y^2y^2 + \frac{1}{2}M\omega_z^2z^2$
- 2D gas: **compress strongly** the transverse direction ( $z$ )  
 $\omega_z \gg \omega_{x,y}$  such that  $\mu, k_B T \ll \hbar\omega_z$ : **frozen along  $z$**
- Ground state of **size**  $a_z = \sqrt{\hbar/M\omega_z}$



**Two cases** depending of the ratio  $a/a_z$  where  $a$  is the **3D scattering length**.

# Case 1: The quasi two-dimensional Bose gas: $a_z > a$

Collisions remain 3D

- $\psi(\mathbf{r}) = \psi_{\perp}(\mathbf{r}_{\perp})\phi_z(z)$  with  $-\hbar^2\partial_z^2\phi + \frac{1}{2}m\omega_z^2z^2\phi(z) = \frac{\hbar\omega_z}{2}\phi(z)$
- Plug into GPE:

$$\psi_{\perp} \left\{ \mu\phi(z) - \left[ -\frac{\hbar^2}{2M}\Delta_z\phi_z + \frac{1}{2}m\omega_z^2z^2\phi_z(z) \right] \right\} =$$

$$\left[ -\frac{\hbar^2}{2M}\Delta_{\perp}\psi_{\perp} + V(\mathbf{r}_{\perp})\psi_{\perp} \right] \phi(z) + g|\psi_{\perp}|^2\psi_{\perp}|\phi(z)|^2\phi(z)$$

- Average over the  $z$  degree of freedom:  $\int \phi(z)^* \times \dots$

$$\text{2D GPE} \quad \boxed{\mu_2\psi_{\perp} = -\frac{\hbar^2}{2M}\Delta_{\perp}\psi_{\perp} + V(\mathbf{r}_{\perp})\psi_{\perp} + g_2|\psi_{\perp}|^2\psi_{\perp}}$$

where  $\boxed{\mu_2 = \mu - \frac{1}{2}\hbar\omega_z}$  and  $g_2 = g \int |\phi(z)|^4 dz = \frac{g}{\sqrt{2\pi}a_z}$

- Dimensionless interaction  $g_2 = \frac{\hbar^2}{M}\tilde{g}$  with  $\boxed{\tilde{g} = \sqrt{8\pi}\frac{a}{a_z}}$

## Case 2: Exotic case: $a_z < a$

True 2D collisions

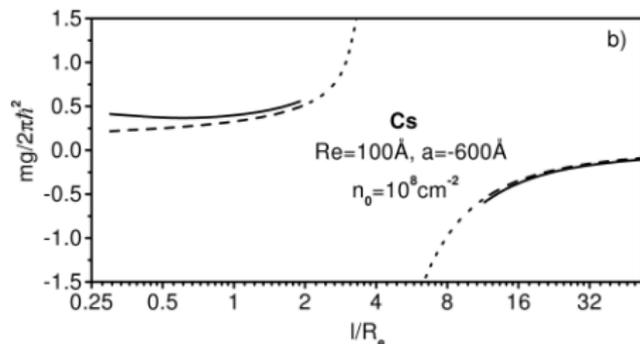
Collisions occur in 2D, scattering length  $a_{2D} \simeq 2a_z e^{-\sqrt{\frac{\pi}{2}} \frac{a_z}{a}} \neq 0$

Renormalization of the interaction constant:  $g_2 \times f\left(\frac{a}{a_z}, na_{2D}^2\right)$

- Coupling 'constant' **depends on atomic density!**
- Modified EOS:  $\mu(n) \neq g_2 n \Rightarrow$  quantum anomaly
- $g_{2D} > 0$  possible for small  $a_z$  even if  $a < 0$

confinement-induced  
resonance for  $a < 0$

[Petrov, Holzmann, Shlyapnikov  
(2000)]



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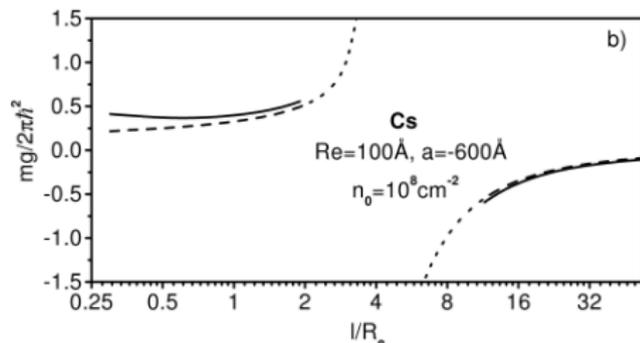
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- Coupling 'constant' **depends on atomic density!**
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In the following we consider essentially **case 1** with  $\tilde{g} = \sqrt{8\pi} a/a_z$ .

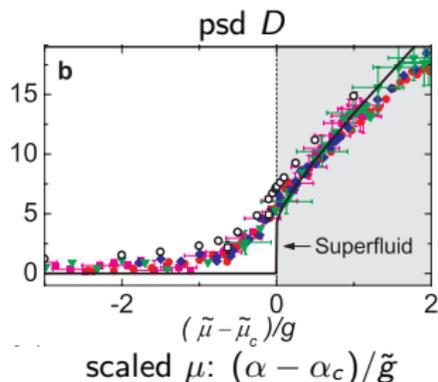
# The two-dimensional Bose gas

## 2D: A marginal dimension

2D is a very special case! **Logs and topological phase transitions**

### • Scaling symmetry and universality

- kinetic energy  $\propto k^2$ , interactions  $\propto 1/r^2$ , integrand  $k dk \Rightarrow$  critical dimension with **Log divergences**
- **no length scale**: dimensionless interaction strength  $g = \frac{\hbar^2}{M} \tilde{g}$
- EOS depends only on  $\alpha = \mu/k_B T$ :  
 $D = f(\alpha, \tilde{g})$  [ENS, Chicago]



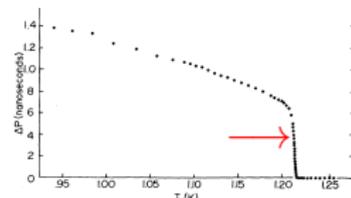
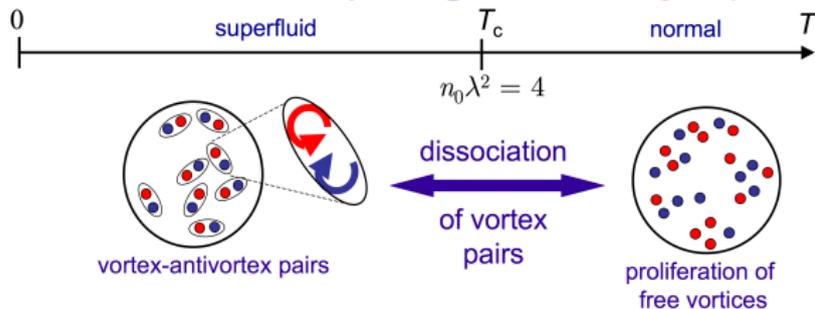
[Chin et al. 2011]

# The two-dimensional Bose gas

## 2D: A marginal dimension

2D is a very special case! **Logs and topological phase transitions**

- **2D homogeneous case** No long range order/BEC (Hohenberg–Mermin–Wagner theorem), but a Kosterlitz–Thouless transition to a superfluid state below  $T_{\text{BKT}}$ , relying on **vortex-antivortex pairing**. **Universal jump** of the superfluid density.



Bishop and Reppy

[ENS-CdF, NIST, Chicago, Palaiseau, Seoul, Cambridge...]

**2016 Nobel prize** in physics to Haldane, Kosterlitz and Thouless

# The two-dimensional Bose gas

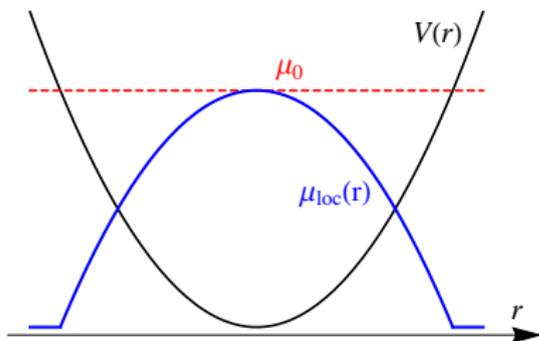
## 2D: A marginal dimension

- **trapped gas**  $V(\mathbf{r})$ :
  - **BEC** recovered in a harmonic trap (finite size helps)
  - **BKT** still relevant within **local density approximation** (LDA).  
**BEC-BKT** interplay [Cambridge]

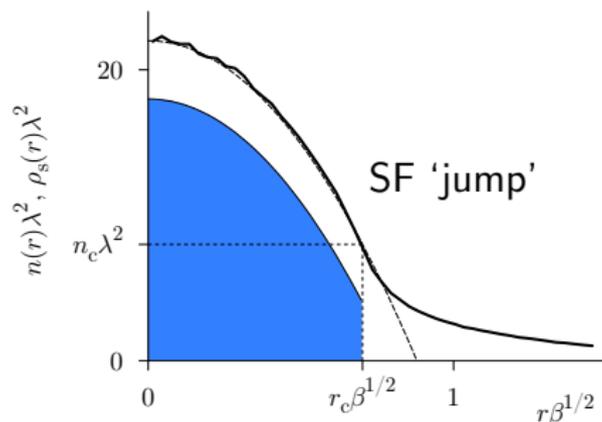
replace

$$\mu \text{ by } \mu_{\text{loc}}(\mathbf{r}) = \mu_0 - V(\mathbf{r}),$$

$$\alpha \text{ by } \alpha_{\text{loc}}(\mathbf{r}) = \alpha_0 - V(\mathbf{r})/k_B T$$



BKT **superfluid phase** within LDA



from Holzmann & Krauth, PRL 2008

# The two-dimensional Bose gas

## 2D: A marginal dimension

2D is a very special case! **Logs and topological phase transitions**

Summary:

	ideal	interacting
homogeneous	no BEC, no SF	<b>BKT</b> SF [ENS-CdF]
trapped	<b>BEC</b> , no SF	<b>BEC</b> + <b>BKT</b> within LDA

# The two-dimensional Bose gas

## 2D: A marginal dimension

2D is a very special case! **Logs and topological phase transitions**

Summary:

	ideal	interacting
homogeneous	no BEC, no SF	<b>BKT</b> SF [ENS-CdF]
trapped	<b>BEC</b> , no SF	$\leftrightarrow$ <b>BEC</b> + <b>BKT</b> within LDA

**BEC-BKT** interplay [Cambridge]

# Experimental implementation

rf-induced adiabatic potentials – the dressed quadrupole trap

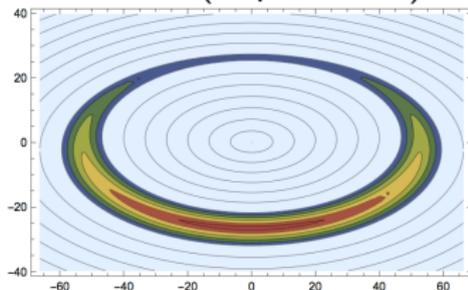
Adiabatic potentials for rf-dressed atoms: **dressed quadrupole trap**

[reviews Garraway/Perrin: JPB 2016 and Adv.At.Mol.Opt.Phys. 2017]

Atoms are confined to an **isomagnetic surface** of a quadrupole field.

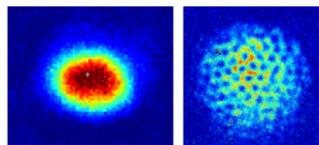
- smooth potentials (magnetic fields with large coils)
- strong confinement to the surface:  $\omega_{\perp} \sim 2\pi \times 1 - 2$  kHz
- geometry ( $r_0$ ,  $xy$ -anisotropy) can be fine-tuned dynamically
- temperature adjusted with a (weak) rf knife (30 – 200 nK)

side view (isopotentials):



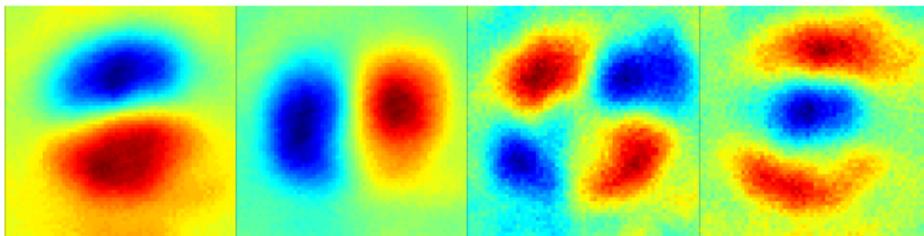
top-view:

a 2D quantum gas



# Collective modes of the quasi-2D Bose gas

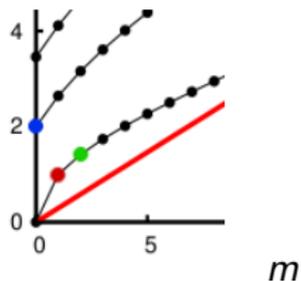
## Overview of the collective modes



# Reminder for a 2D trapped Bose gas

## Excitation spectrum and collective modes

Collective modes for the isotropic 2D gas:  $n, m$  are good quantum numbers:  $\omega(n, m) = \omega_0 [2n^2 + 2n|m| + 2n + |m|]^{1/2}$

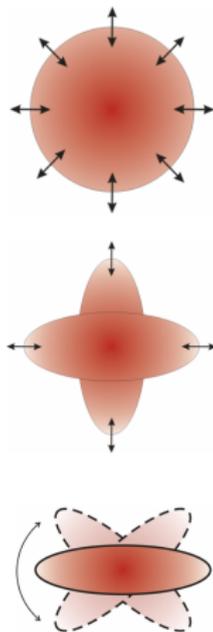


- **dipole** mode  
 $n = 0, m = 1$ , both  
superfluid and thermal:  
centre of mass  
oscillation: **clock**

- **monopole**  $n = 1, m = 0$ :  
superfluid and thermal  
**signature of the EOS**

- **quadrupole**  $n = 0, m = \pm 2$   
**signature of superfluidity**

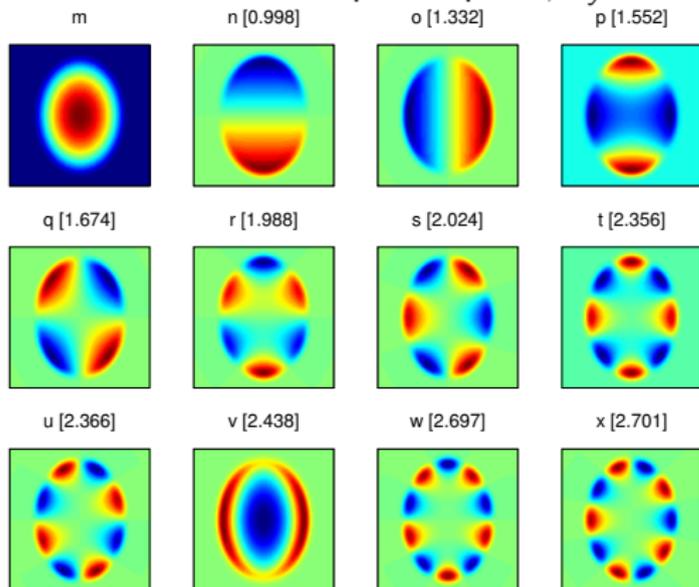
- **scissors** for  $\omega_x \neq \omega_y$   
**signature of superfluidity**



# Expected collective modes in an anisotropic trap

From Bogolubov diagonalisation of an idealised case

Bogolubov modes computed numerically for the 2D gas in a harmonic anisotropic trap  $\omega_x, \omega_y$ :



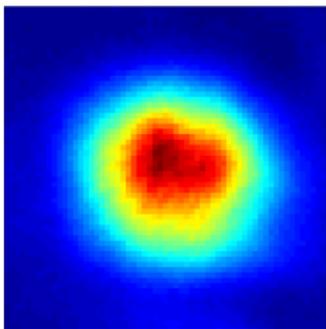
2 dipoles ( $\omega_x, \omega_y$ ),  
 quadrupole-like ( $\omega_Q$ ),  
 scissors  
 ( $\omega_S = \sqrt{\omega_x^2 + \omega_y^2}$ ), 4 more  
 modes of higher order  
 symmetry and then  
 monopole-like ( $\omega_M$ )

# Overview of low energy collective modes

## Exciting low energy collective modes

A BEC prepared in 3D trap and transferred quickly into the 2D rf-dressed quadrupole trap, whose axes are also suddenly rotated.

Several modes are excited during this process.



excited cloud

2D trap frequencies:

$$\omega_x = 2\pi \times 33 \text{ Hz},$$

$$\omega_y = 2\pi \times 44 \text{ Hz}$$

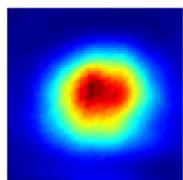
133 images

taken during 100 ms, after various holding times.

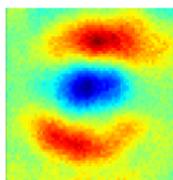
# Overview of the Bogolubov modes

## Principal component analysis

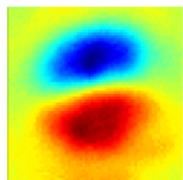
Analysis of the **correlations between pixels** allows to recover the collective modes.



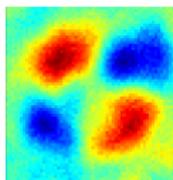
average picture



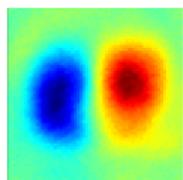
monopole-like



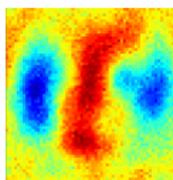
dipole mode  $x$



scissors



dipole mode  $y$



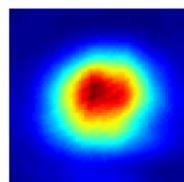
quadrupole-like

R. Dubessy et al., Fast Track Comm. of New J. Phys. **16**, 122001 (2014) + video abstract.

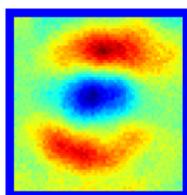
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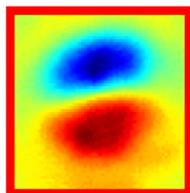
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average picture



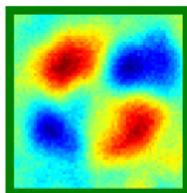
**monopole-like**  
EOS



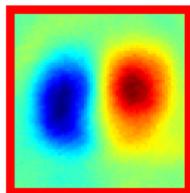
dipole mode x

**serve as a clock**

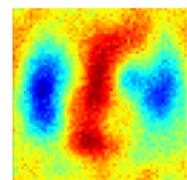
for  $\omega_x, \omega_y$



**scissors**  
superfluidity



dipole mode y

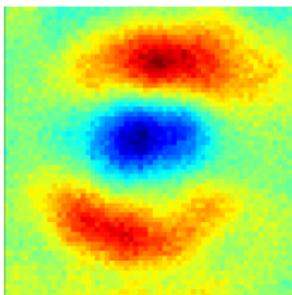


quadrupole-like

R. Dubessy et al., Fast Track Comm. of New J. Phys. **16**, 122001 (2014) + video abstract.

# The monopole mode

## Monopole mode and Equation of State



$$\mu(n) \propto n^\gamma, \gamma = ?$$

# The monopole mode in an isotropic harmonic trap

A way to study the Equation Of State

isotropic harmonic 2D trap, frequency  $\omega$

- monopole probes the **compressibility**  $\Rightarrow \Omega_M$  is related to the 2D EOS  $\mu(n)$ :

$$\Omega_M = \sqrt{2(2 + \epsilon)} \omega \quad \text{with} \quad \epsilon = \frac{n\mu''(n)}{\mu'(n)}$$

cf Rudi Grimm's expt with fermions [Altmeyer 2006]

- Ex: 2D weakly interacting gas:  $\mu(n) = gn \Rightarrow \Omega_M = 2\omega$

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- Ex: **quantum anomaly** due the beaking of scaling symmetry:  $\tilde{g}/(16\pi)$  **positive shift** [Olshanii 2010]

# The monopole mode in an isotropic harmonic trap

Quantum anomaly

Olshanii, Perrin, Lorent, PRL 2010

isotropic harmonic 2D trap, frequency  $\omega$

**Pitaevskii-Rosch symmetry: Classical Field Theory (CFT) for 2D bosons in a harmonic trap**

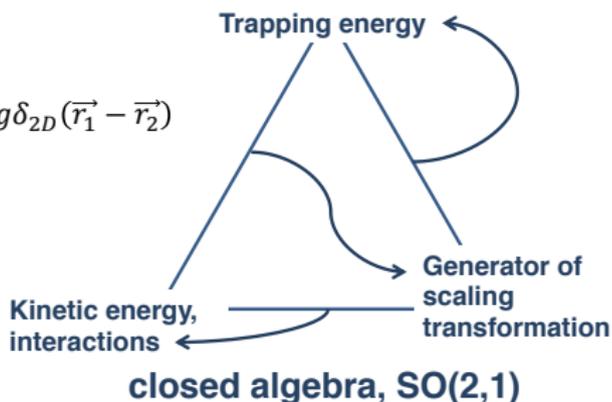
**Scaling invariance** in 2D in the CFT:  $\hat{H}_0 = \hat{H}_K + \hat{H}_I$ ,  $\hat{H}_{\text{trap}}$ ,  $\hat{Q}$  form a closed algebra  $SO(2,1)$

Kinetics, interactions

$$\begin{cases} \frac{p^2}{2m} \xrightarrow{\vec{r} \rightarrow \lambda \vec{r}} \frac{1}{\lambda^2} \frac{p^2}{2m} \\ g \delta_{2D}(\vec{r}_1 - \vec{r}_2) \xrightarrow{\vec{r} \rightarrow \lambda \vec{r}} \frac{1}{\lambda^2} g \delta_{2D}(\vec{r}_1 - \vec{r}_2) \end{cases}$$

trap:  $\frac{1}{2} M \omega^2 r^2 \xrightarrow{\vec{r} \rightarrow \lambda \vec{r}} \lambda^2 \frac{1}{2} M \omega^2 r^2$

generator of scaling transformations:  $\frac{1}{2} (\mathbf{r} \cdot \mathbf{p} + \mathbf{p} \cdot \mathbf{r})$



# The monopole mode in an isotropic harmonic trap

Quantum anomaly

Olshanii, Perrin, Lorent, PRL 2010

Petrov 2001:  $a_{2D} = 1.48 \dots a_{1D} \exp\left[-\frac{\sqrt{\pi}}{2} \frac{a_{1D}}{a_{3D}}\right]$  a quantum length scale appears!

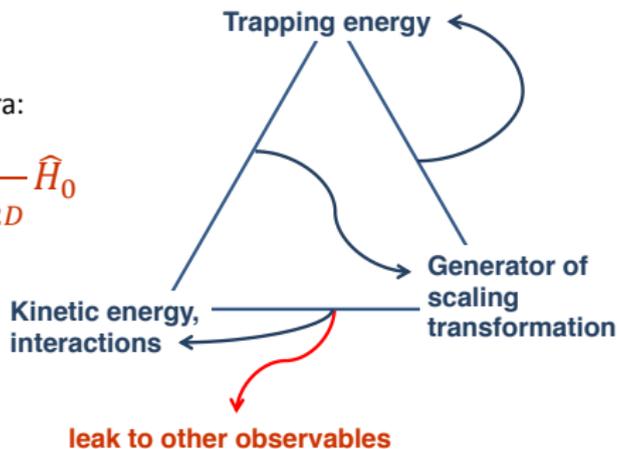
New equation of state:  $\mu(n) = \frac{4\pi\hbar^2}{m} n \chi(\pi e^{2\gamma+1} n a_{2D}^2)$  (Popov 1983, Mora&Castin 2003)

where  $\chi(x) = \frac{1}{-W_{-1}(-x)} \underset{x \rightarrow 0}{\approx} 1/\ln(1/x) + \mathcal{O}\left(\frac{\ln(\ln(1/x))}{\ln(1/x)^2}\right)$

Consequence: 'leak' in the algebra:

$$[\hat{Q}, \hat{H}_0] = 2i\hat{H}_0 + ia_{2D} \frac{\partial}{\partial a_{2D}} \hat{H}_0$$

$\Rightarrow$  small shift of  $\Omega_M$



# The monopole mode in an isotropic harmonic trap

A way to study the Equation Of State

isotropic harmonic 2D trap, frequency  $\omega$

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cf Rudi Grimm's expt with fermions [Altmeyer 2006]

- Ex: 2D weakly interacting gas:  $\mu(n) = gn \Rightarrow \Omega_M = 2\omega$
- Ex: **quantum anomaly** due the beaking of scaling symmetry:  $\tilde{g}/(16\pi)$  **positive shift** [Olshanii 2010]
- Ex: flat, but **3D gas**:  $\mu(n) \propto n^{2/3} \Rightarrow \Omega_M = \sqrt{10/3}\omega$

# The monopole mode in an isotropic harmonic trap

## A way to study the Equation Of State

isotropic harmonic 2D trap, frequency  $\omega$

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- Ex: flat, but **3D gas**:  $\mu(n) \propto n^{2/3} \Rightarrow \Omega_M = \sqrt{10/3}\omega$
- we probe the **intermediate case**: for non negligible **interactions** is there a shift  $a$  as function of  $\frac{\mu}{2\hbar\omega_z}$ ? [Merloti 2013]

# Observation of the monopole mode

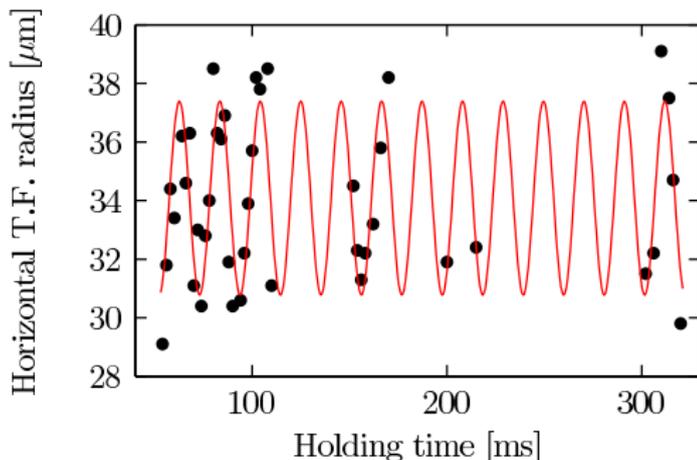
## Isotropic trap

Prepare a degenerate sample in an **isotropic 2D trap**

Excitation through a sudden change in  $\omega$

Very low  $T$  (no thermal fraction)

- experimental data
  - sinusoidal fit
- [Merloti NJP2013]

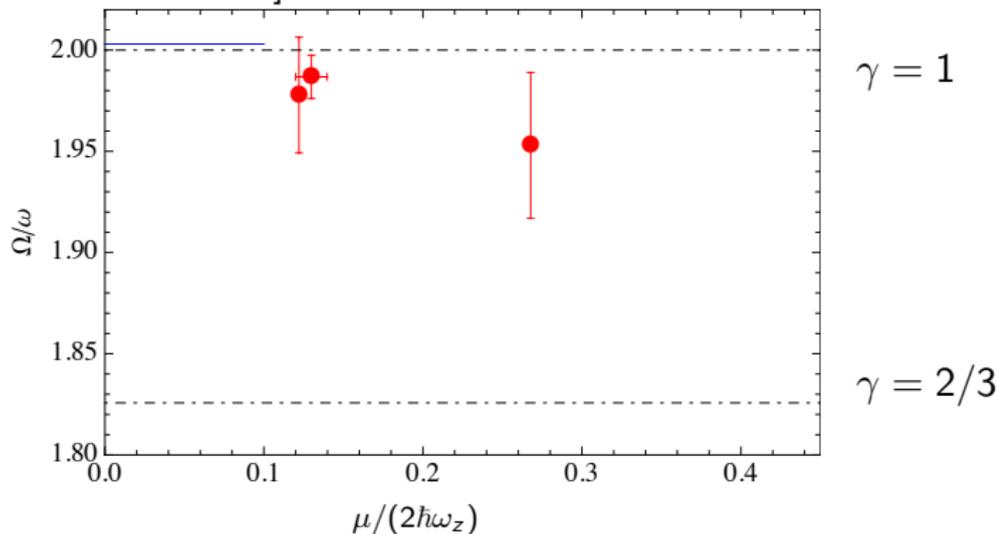


typical data:  $\Omega_M$  close to  $2\omega$ ; **no measurable damping**

# Results: shift of the monopole mode

## A modified EOS

We observe a small **negative shift** as a function of  $\mu/(2\hbar\omega_z)$   
 [Merloti PRA2013]:



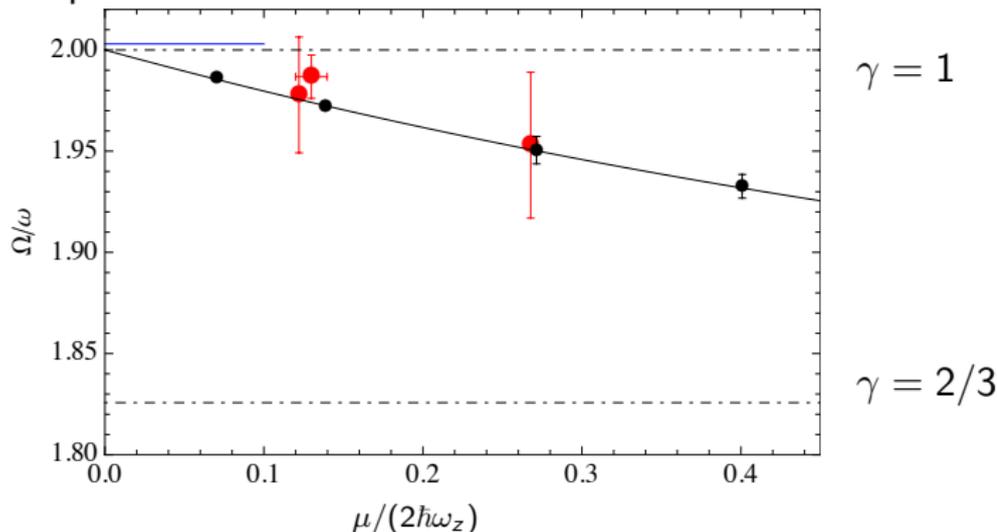
The finite  $z$  frequency implies a **modified EOS**.

Typically 1% shift:  $\gamma = 0.96$ ,  $\mu \propto n^{0.96}$ .

# Results: shift of the monopole mode

## A modified EOS

Comparison with a **3D** GPE simulation:

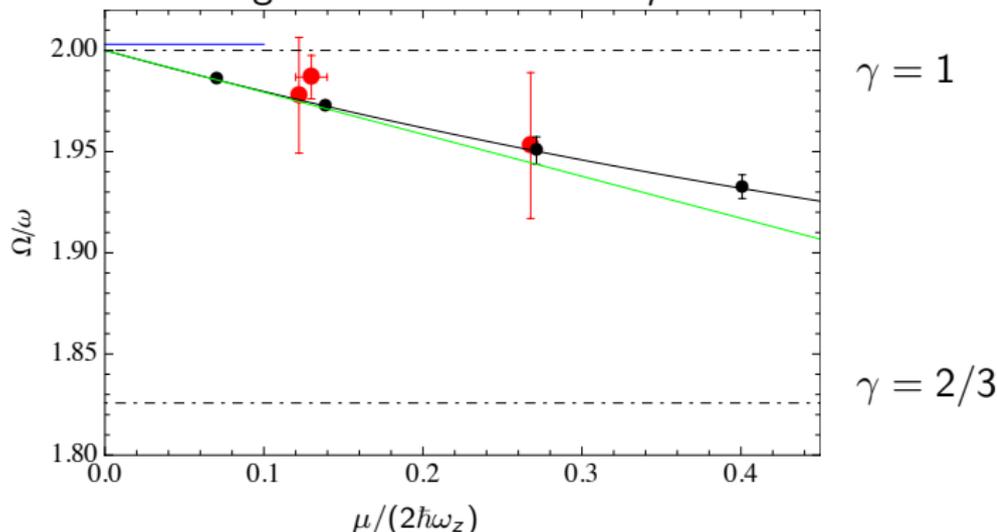


The in-plane EOS is indeed impacted by the **third dimension**.

# Results: shift of the monopole mode

## A modified EOS

Comparison with a **perturbative theory** (Olshanii): interactions deform the 1D ground state and shift  $\mu$ .



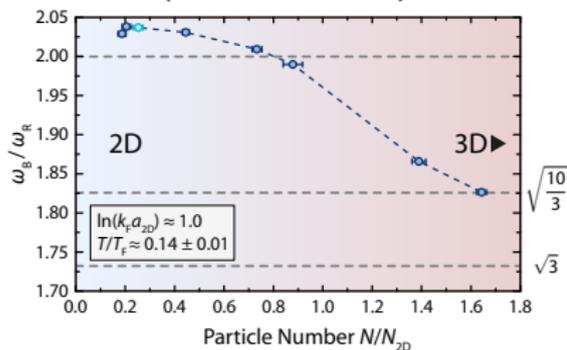
Recover the observed behaviour at first order.

Merloti et al., PRA **88**, 061603(R) 2013.

# Observing the quantum anomaly?

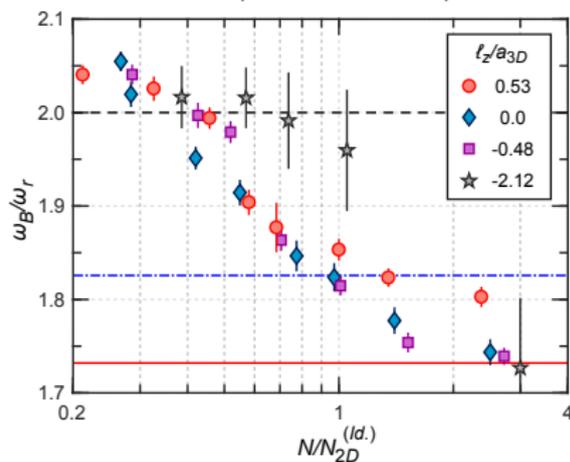
Recently observed **with fermions**, using a Feshbach resonance

S. Jochim, Heidelberg,  ${}^6\text{Li}$   
(also uses PCA)



[PRL **121**, 120401 (2018)]

Chris Vale, Swinburne,  ${}^6\text{Li}$

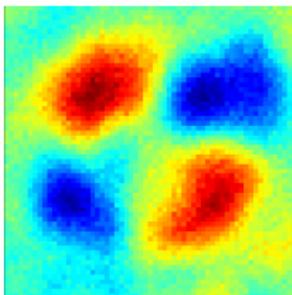


[PRL **121**, 120402 (2018)]

For **bosons**: yet to be done

# The scissors mode

## Scissors mode and superfluid transition

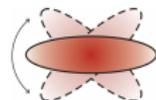


thermal or superfluid gas?

# The scissors mode

A signature of superfluidity in a dilute 2D gas

Using the scissors mode to characterize a **superfluid** dilute gas [DGO Stringari 1999, Foot2000]

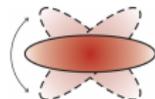


**Scissors mode**: oscillation of  $\langle xy \rangle \propto \theta$  in an **anisotropic** harmonic trap,  $\omega_x/\omega_y \sim 1.3$

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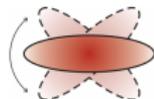
**Scissors mode**: oscillation of  $\langle xy \rangle \propto \theta$  in an **anisotropic** harmonic trap,  $\omega_x/\omega_y \sim 1.3$

- scissors mode expected at  $\omega_{sc} = \sqrt{\omega_x^2 + \omega_y^2}$  for a **superfluid**
- no scissors mode in the thermal phase in the **collisionless** regime, only beat notes of harmonic modes  $\omega_{\pm} = \omega_x \pm \omega_y$

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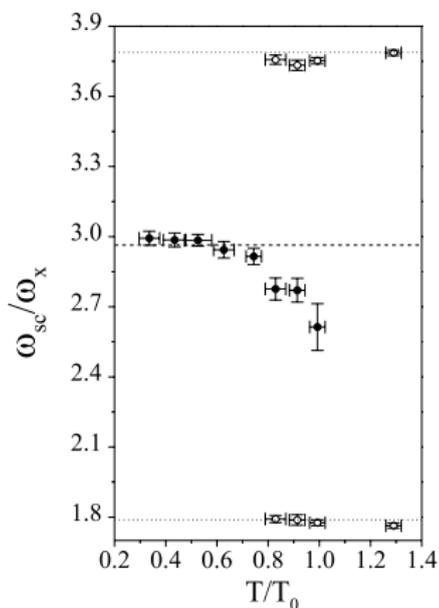
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- no scissors mode in the thermal phase in the **collisionless** regime, only beat notes of harmonic modes  $\omega_{\pm} = \omega_x \pm \omega_y$
- crossover between the two regimes when  $T$  increases?

$\Rightarrow$  Use the scissors mode as a **signature of superfluidity** of a dilute gas across the BKT transition!

# The scissors mode: previous work and expectations

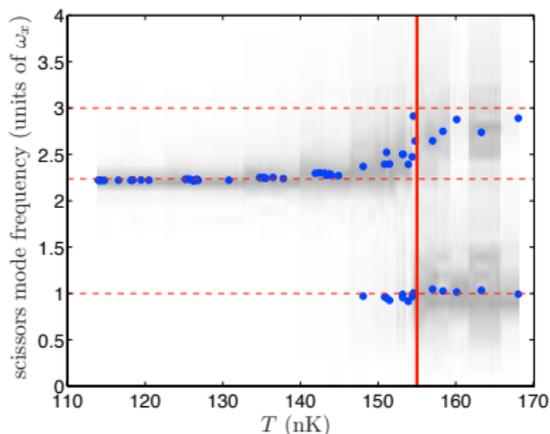
## 3D vs 2D as a function of temperature

3D, bimodal fit of the angle(s):  
observed negative shift



exp: [Marago, Foot PRL 2001]

2D: **positive shift** expected  
 $\omega_{sc}$  connects to  $\omega_+$

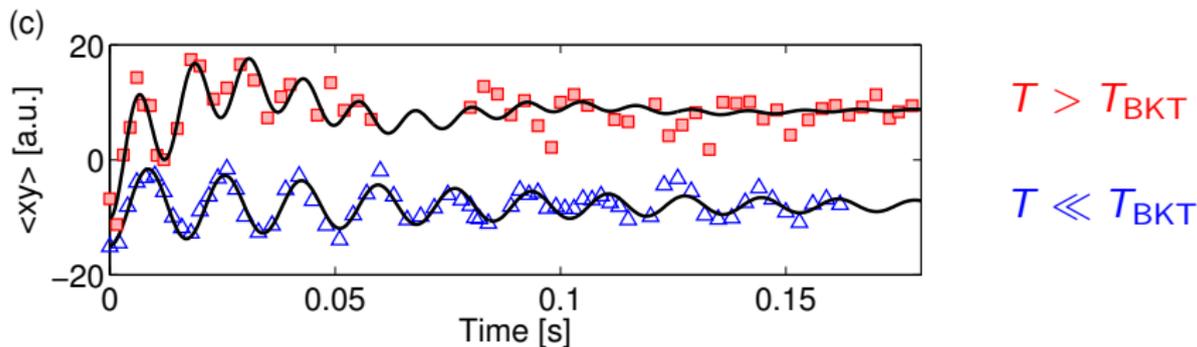


numerics: [Simula, PRA 2008]  
compute  $\langle xy \rangle$ , FFT analysis

# Exciting the scissors mode

Procedure:

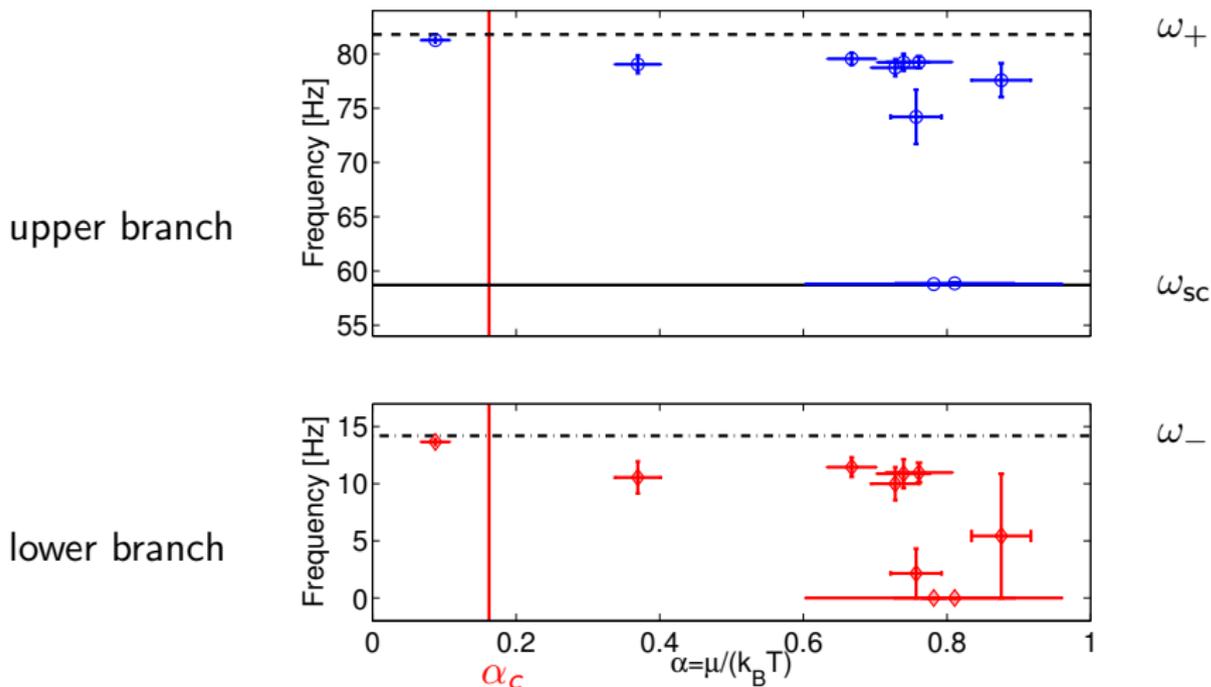
- Anisotropic trap + sudden rotation of the trap axes.
- Compute  $\langle xy \rangle$  and plot its time variation.
- Extract oscillation frequency  $\omega$  and damping  $\Gamma$ .
- Repeat for various  $\mu$  and  $T$  (i.e.  $\alpha = \mu/k_B T$ )



# Results with a global analysis of $\langle xy \rangle$

Two frequency branches: **upper branch** from  $\omega_+$  to  $\omega_{SC}$ , **lower branch** from  $\omega_-$  to 0.

**BKT**



# Results with a global analysis of $\langle xy \rangle$

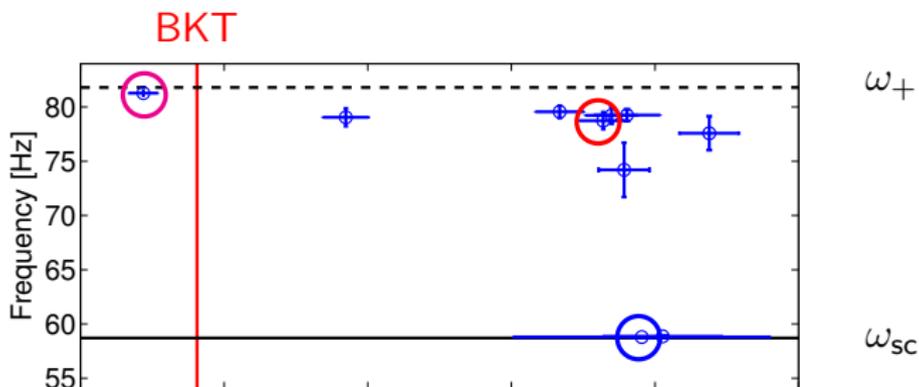
Two frequency branches: **upper branch** from  $\omega_+$  to  $\omega_{SC}$ , **lower branch** from  $\omega_-$  to 0.

$\alpha \gg \alpha_c$

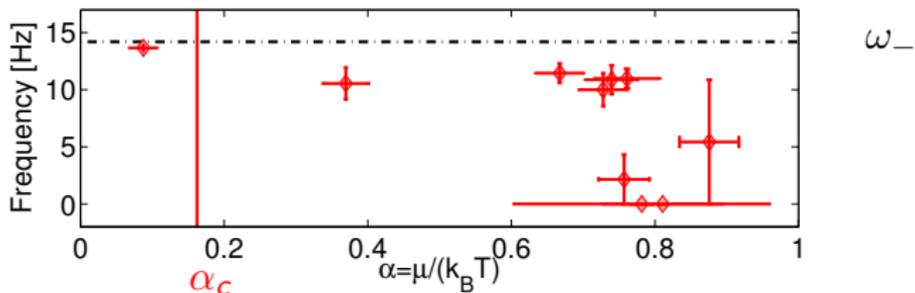
$\alpha > \alpha_c$

$\alpha < \alpha_c$

upper branch

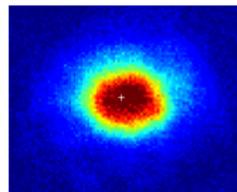


lower branch



# Going local

The frequencies  $\omega_{\pm}$  are present in  $\langle xy \rangle(t)$  even for  $\alpha > \alpha_c$  (or  $T < T_{\text{BKT}}$ ), where a superfluid should be present.



The gas is inhomogeneous...

- Superfluid oscillation hidden by thermal contribution to  $\langle xy \rangle$ ?
- Can we get **more local information**?
- Can we identify the superfluid phase in the inhomogeneous gas with **purely dynamical criteria**?

⇒ perform a **local analysis** of the dynamics

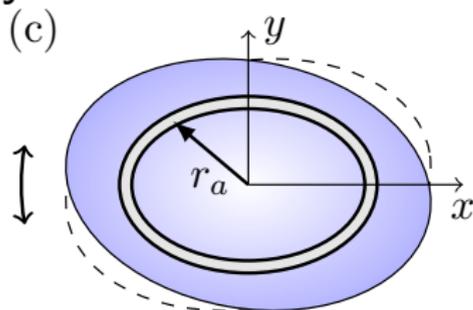
# Analysing the local average

**Local analysis:** use the fact that the scissors oscillation is a **surface mode**

In the spirit of LDA, compute the  $\langle xy \rangle_{r_a}$  average over an annulus, isopotential of **given average density**

rescaled radius  $r_a$ :

$$\omega_x^2 x^2 + \omega_y^2 y^2 = \omega_0^2 r_a^2$$



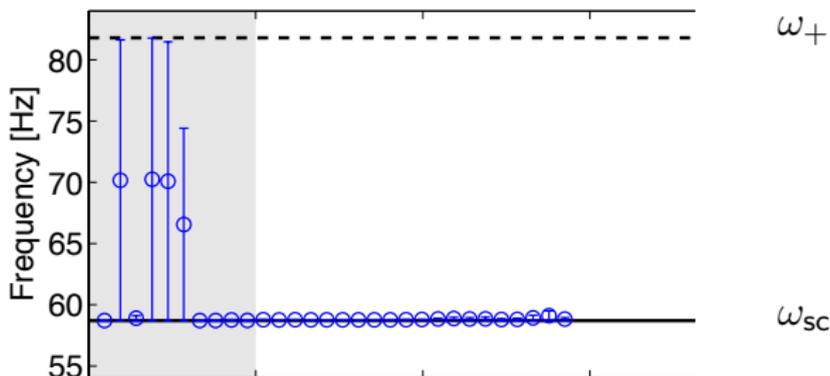
Extract the local values of  $\omega$  and  $\Gamma$

Three cases  $\alpha \gg \alpha_c$ ,  $\alpha > \alpha_c$ ,  $\alpha < \alpha_c$

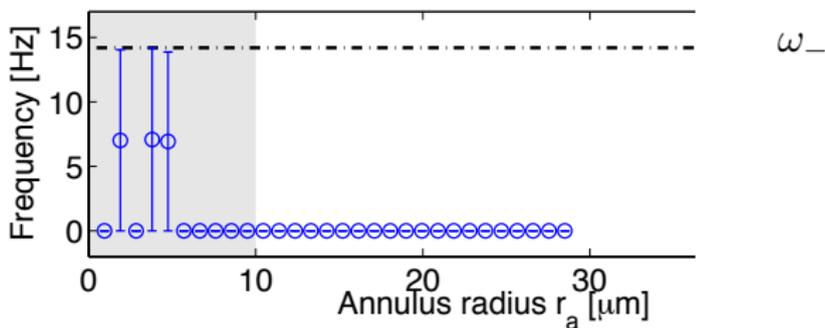
## Analysing the local average

$$\alpha \gg \alpha_c$$

upper branch



lower branch



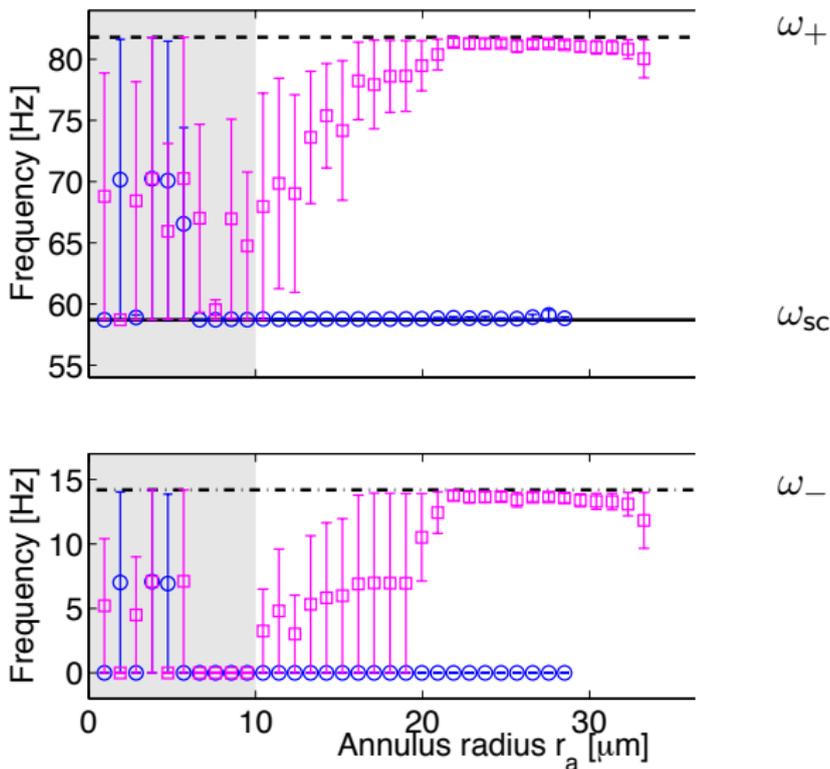
# Analysing the local average

$$\alpha \gg \alpha_c$$

$$\alpha < \alpha_c$$

upper branch

lower branch



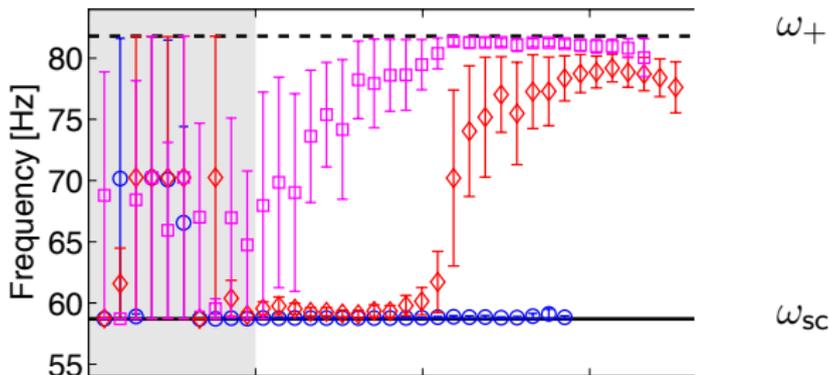
## Analysing the local average

$$\alpha \gg \alpha_c$$

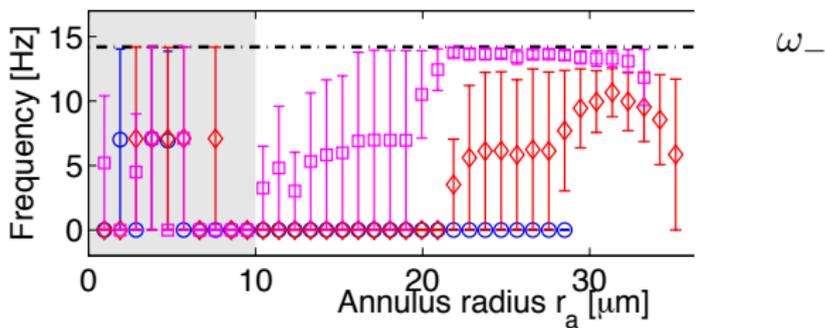
$$\alpha > \alpha_c$$

$$\alpha < \alpha_c$$

upper branch



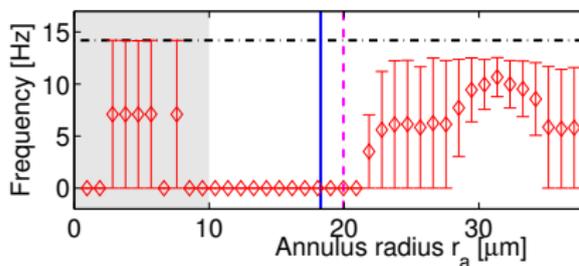
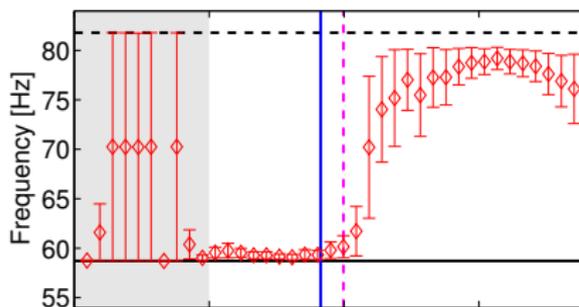
lower branch



# Comparison with BKT LDA threshold

Case  $\alpha > \alpha_c$

BKT ( $\alpha_{\text{loc}} = \alpha_c$ )



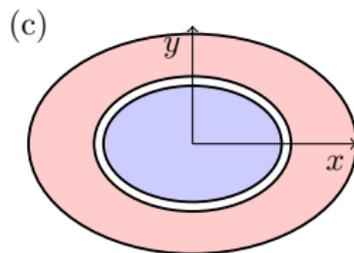
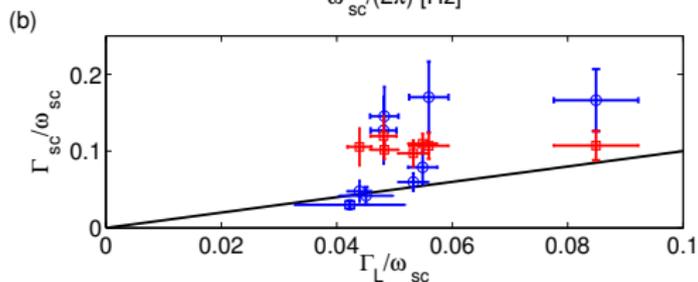
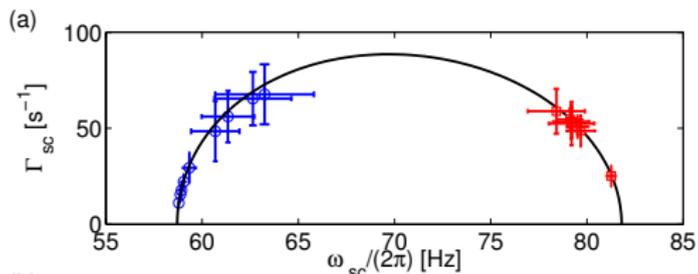
The equilibrium LDA threshold for BKT is in agreement with the local analysis of the dynamics.

[De Rossi et al., NJP 2016]

Open question: can we use it to determine the SF fraction?

# Superfluid - normal boundary

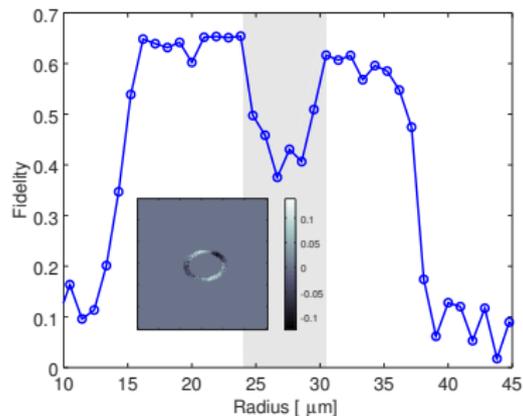
**Conclusion:** the superfluid-normal boundary is located with a **purely dynamical criterion** = frequency of the scissors mode. Damping analysis on each side of the boundary: larger than Landau damping  $\Rightarrow$  SF to thermal gas coupling?



boundary determined with LAA

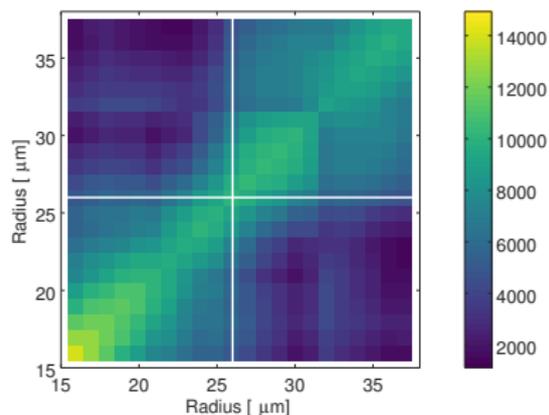
# Bonus: Local PCA

PCA applied on an annulus also reveals the scissors mode 



boundary determined by comparison  
between PCA eigenvectors and the  
 $\langle xy \rangle$  mode

[Dubessy 2018]

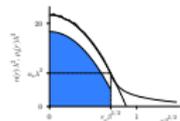
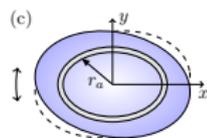
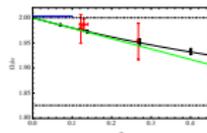
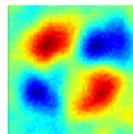


correlations between two radii:  
qualitative agreement with a  
**two-fluid model**

# Summary & prospects

**2D Bose gas:** a very smooth and tunable trap to study the collective modes

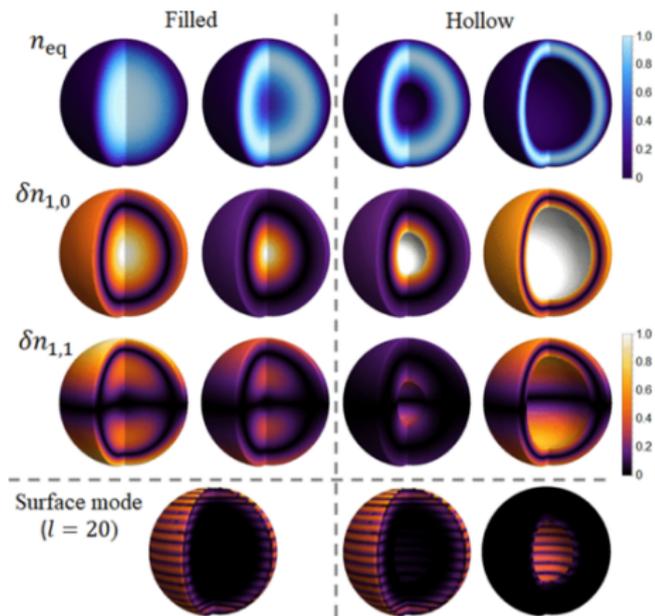
- Direct observation of the mode shapes
- A modified EOS evidenced with the **monopole** mode
- The **scissors mode** reveals normal-to-superfluid boundary with a **local** analysis of the **dynamics**.
- Outlook: use this probe to access a sharp change in  $\rho_s$  at the boundary?



# Summary & prospects

## Beyond the bottom of the bubble:

- Looking for the collective modes of a shell (cf Natália's talk)

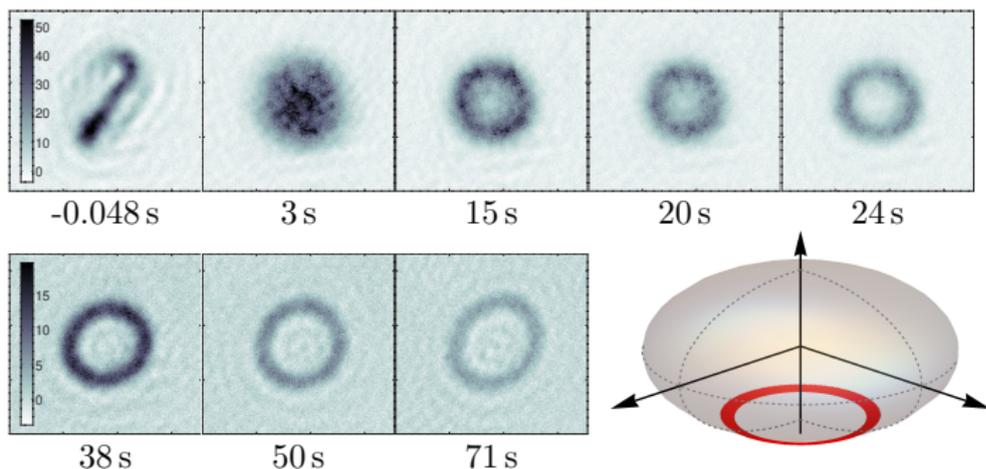


Sun et al., Phys. Rev. A **98**, 013609 (2018)]

# Summary & prospects

## Beyond the bottom of the bubble:

- Observation of the collective modes of a shell
- Fast rotation in the shell: **superfluid supersonic flow**



[Guo et al., submitted]

# Acknowledgments



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L. Longchambon

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