An example of a quantum anomaly in the physics of cold atomic gases

Maxim Olshanii, Hélène Perrin, and Vincent Lorent

Department of Physics, University of Massachusetts Boston

Laboratoire de Physique des Lasers, Université Paris 13 and CNRS.

Summary

We propose an experimental scheme for the observation of a quantum anomaly — quantum-mechanical symmetry breaking — in a 2D harmonically trapped Bose gas. The anomaly manifests itself in a shift of the monopole excitation frequency away from the value dictated by the Pitaevskii-Rosch dynamical symmetry. While the corresponding classical Gross-Pitaevskii equation and the derived from it hydrodynamic equations do exhibit this symmetry, it is violated under quantization. The resulting frequency shift is of the order of 1% of the carrier, well in reach for modern experimental techniques. We propose using the dipole oscillations as a frequency gauge.

Introduction

What is a quantum anomaly?

Classical theory with a symmetry → canonical quantization → Quantum divergent theory, symmetry preserved → regularization → Quantum-regular theory, symmetry **BROKEN**

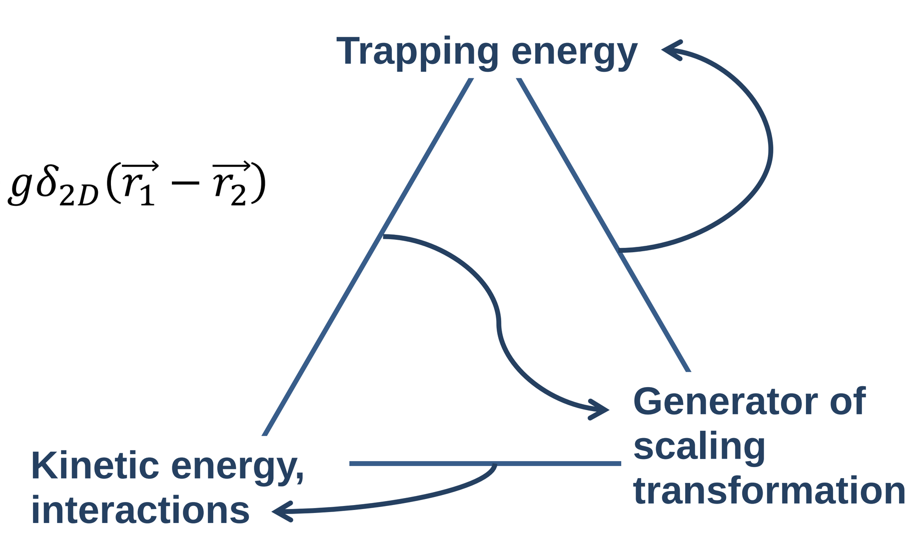
Pitaevskii-Rosch symmetry: Classical Field Theory (CFT) for 2D bosons in a harmonic trap

Scaling invariance in 2D in the CFT: $\hat{H}_0 = \hat{H}_K + \hat{H}_I$, $\hat{H}_{\rm trap}$, $\hat{Q}$ form a closed algebra SO(2,1)

Kinetics, interactions:
$$\begin{cases} \frac{p^2}{2m} \xrightarrow{\vec{r}\to\lambda\vec{r}} \frac{1}{\lambda^2}\frac{p^2}{2m} \\ g\delta_{2D}(\vec{r}_1-\vec{r}_2) \xrightarrow{\vec{r}\to\lambda\vec{r}} \frac{1}{\lambda^2} g\delta_{2D}(\vec{r}_1-\vec{r}_2) \end{cases}$$

trap: $\frac{1}{2}M\omega^2 r^2 \xrightarrow{\vec{r}\to\lambda\vec{r}} \lambda^2 \frac{1}{2}M\omega^2 r^2$

generator of scaling transformations: $\frac{1}{2}(\mathbf{r}\cdot\mathbf{p}+\mathbf{p}\cdot\mathbf{r})$



Manifestation of the symmetry: amplitude-independent frequency of $2\omega_{HO}$ of the monopole mode, absence of damping for this mode. (Dalibard 2002)

Other examples: Classical: $1/r^2$ particles, 2D bosons (CFT)
Quantum: 1D hard–core bosons, 3D unitary fermions, $1/r^2$ particles

What about 2D bosons in QFT?

Quantum anomaly in 2D bosons

$\delta_{2D}(\vec{r})$ ill-defined, interaction energy diverges ⇒ regularization is required.

Popov 1983: $\Psi(\vec{r}_1,\cdots,\vec{r}_i,\cdots,\vec{r}_j,\cdots,\vec{r}_N) \propto \ln(|\vec{r}_i-\vec{r}_j|/a_{2D})$, $\quad \vec{r}_i-\vec{r}_j \to \vec{0}$

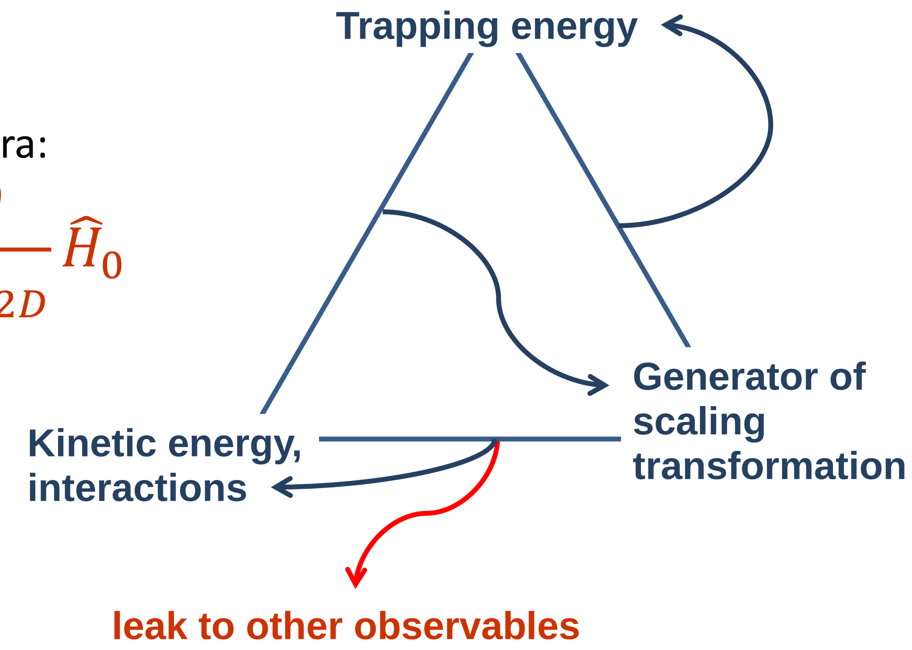
Petrov 2001: $a_{2D} = 1.48\ldots\, a_\perp \exp\left[-\frac{\sqrt{\pi}}{2}\frac{a_\perp}{a_{3D}}\right]$ a quantum length scale appears!

New equation of state: $\mu(n) = \frac{4\pi\hbar^2}{m} n\chi(\pi e^{2\gamma+1} n a_{2D}{}^2)$ (Popov 1983, Mora&Castin 2003)

where $\chi(x) = \frac{1}{-W_{-1}(-x)} \overset{x\to 0}{\approx} 1/\ln(1/x) + \mathcal{O}\left(\frac{\ln(\ln(1/x))}{\ln(1/x)^2}\right)$

Consequence: `leak' in the algebra:

$$[\hat{Q},\hat{H}_0] = 2i\hat{H}_0 + i a_{2D}\frac{\partial}{\partial a_{2D}}\hat{H}_0$$



Shift of the monopole frequency

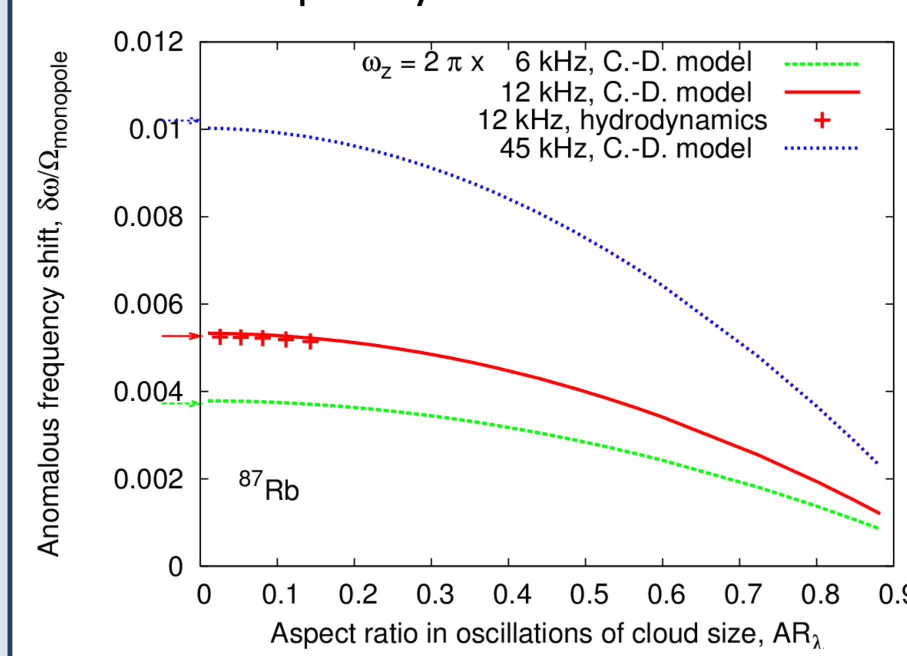
Empirical manifestation of the anomaly: for 2D bosons in a harmonic trap the frequency Ω of *small-amplitude monopole excitation* is shifted as

$$\Omega = 2\omega + \delta\omega \quad \text{where} \quad \frac{\delta\omega}{2\omega} \approx \frac{1}{4\sqrt{\pi}}\frac{a_{3D}}{a_\perp} \quad \text{in the limit } a_{3D} \ll a_\perp.$$

- a_{3D} : 3D scattering length
- $a_\perp = \sqrt{\frac{2\hbar}{m\omega_\perp}}$: transverse confinement length
- $\omega_\perp$: transverse confinement frequency

Order of magnitude for Rb: 0.5% for a transverse confinement frequency 12 kHz.

relative frequency shift for various transverse frequencies and aspect ratios:

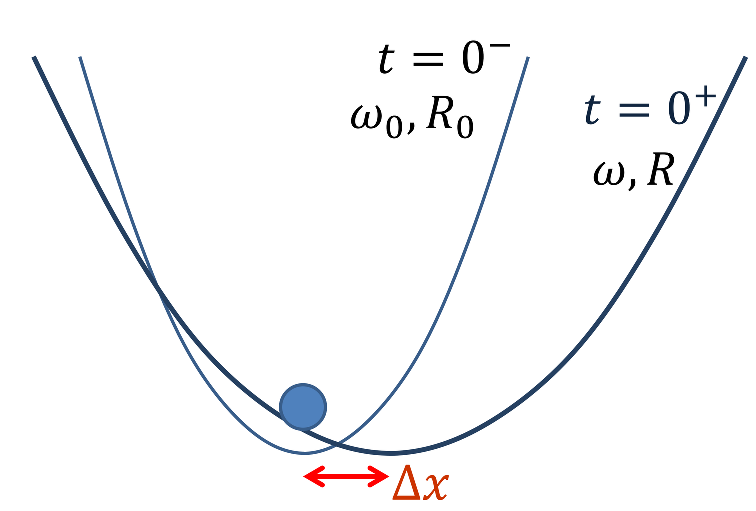


Castin-Dum ansatz:
$$n(\mathbf{r},t) = \frac{1}{\lambda(t)^2} n_0 \left[1 - \left(\frac{r}{\lambda(t)R}\right)^2\right]$$

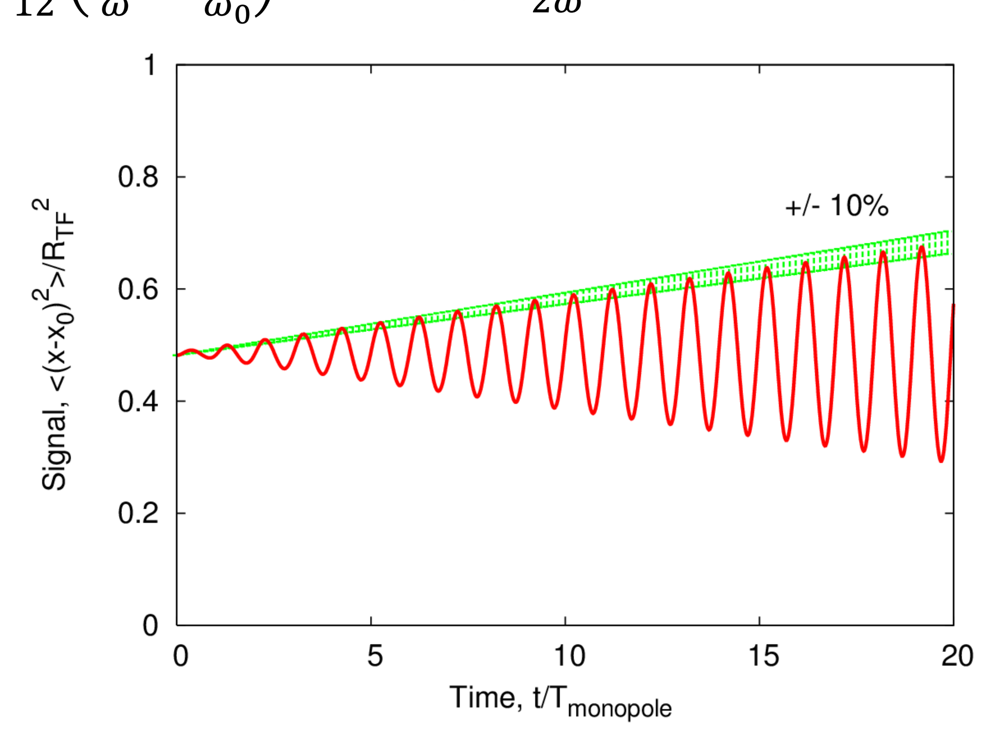
The cloud size oscillates between $R\lambda_{\min}$ and $R\lambda_{\max}$. Aspect ratio AR_λ is defined as $AR_\lambda = \frac{\lambda_{\max}-\lambda_{\min}}{\lambda_{\max}+\lambda_{\min}} = \frac{\lambda_{\max}^2-1}{\lambda_{\max}^2+1} = \frac{\omega_0-\omega}{\omega_0+\omega}$. Corresponding anomaly at this amplitude: $\delta_\lambda = \frac{\delta\omega}{2\omega}$.

Detection with a beat note

- Excite both the dipole (at ω) and monopole modes: Δx and $\omega < \omega_0$.
- Measure $\langle x^2\rangle$: it provides a beat note between Ω and 2ω, at $\delta\omega$.
- With proper choice for Δx, start with a node in the beating amplitude.
- The initial slope of the envelope is $\frac{\delta_\lambda}{12}\left(\frac{\omega_0}{\omega}-\frac{\omega}{\omega_0}\right)$, giving $\delta_\lambda = \frac{\delta\omega}{2\omega}$.



optimal shift to start with a node:
$$\Delta x = \frac{1}{\sqrt{6}}\sqrt{\left(\frac{\omega_0}{\omega}\right)^2 - 1}\, R_0$$

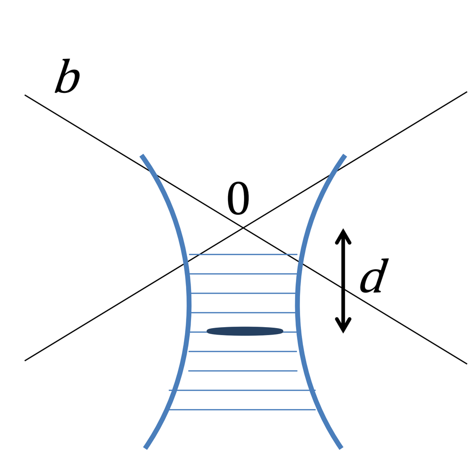


simulation of GPE parameters: $\frac{\omega_\perp}{2\pi} = 45$ kHz, $AR_\lambda = 0.486$

We deduce δ_λ from the fit of the initial slope of the envelope. We get $\delta_\lambda = 0.0076$, to compare with the exact value of $\delta_\lambda = 0.00765618$.

Proposed experimental implementation

2D trap: quadrupole trap of vertical axis + vertical blue-detuned lattice. Load atoms in a well below the magnetic zero. Anisotropy should be avoided (see paper for an estimate). Excitation by a change in magnetic field bias and gradient.



Parameters for ^{87}Rb in its F=2, m_F=2 state

b=167 G/cm
P=4 W at 532 nm focused on w_0=500 µm
d=400 µm (B_0 =13.4 G)
$\frac{\omega}{2\pi}$=41 Hz, $\frac{\omega_\perp}{2\pi}$=45 kHz

References

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